Free boundary equilibrium codes

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Overview

1. Free boundary equilibrium problem

2. Research work
   - Complex poloidal circuits
   - Average quantities over closed flux loop

Grad-Schafranov equation

- Ideal MHD equilibrium satisfies balance

Ideal MHD

$$\nabla p = j \times B \tag{1}$$

- Two-dimensional nonlinear partial differential equation about ψ.

Grad-Schafranov equation

$$\Delta^* \psi + \mu_0 R^2 \frac{\partial}{\partial \psi} p(R, \psi) + g \frac{\partial g}{\partial \psi} = 0 \tag{2}$$

Free boundary problem

- ψ at any point of computational box via equation

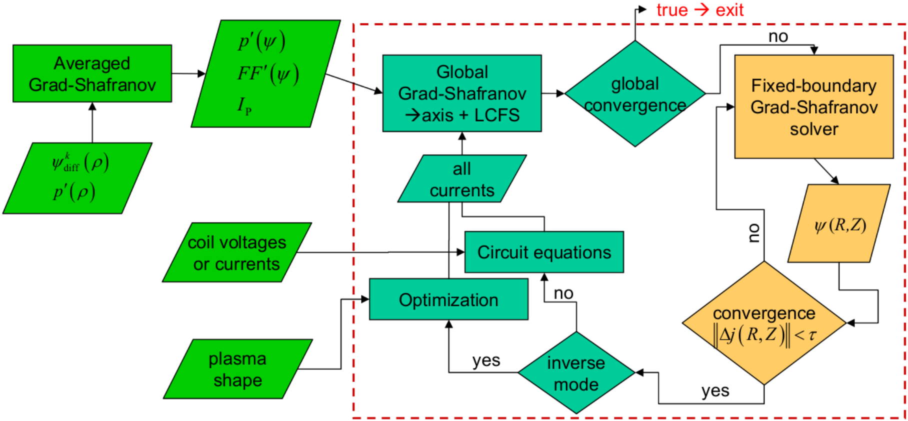
$$\psi(R', Z') = \int_P G(R, Z; R', Z') J_\phi dRdZ + \sum_{i=1}^{N_c} G(R_i^c, Z_i^c; R', Z') I_\phi$$

- J_ϕ may be obtained from ψ

$$J_\phi = -\frac{1}{\mu_0 R} \Delta^* \psi$$

- ψ is result of G-S equation (2)
- In order to solve G-S eq., boundary conditions ψ_b are necessary $\Rightarrow$ Initial guess of ψ_b

FREEBIE scheme



Flow chart of FREEBIE code,[Urban,2012]

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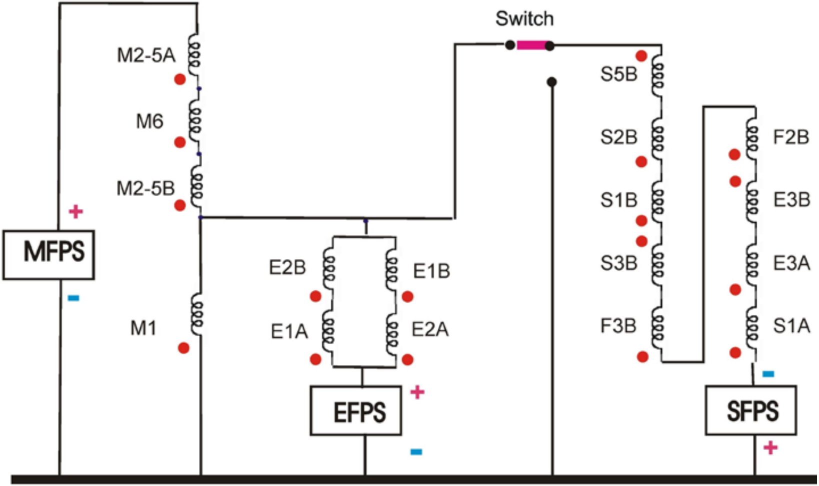
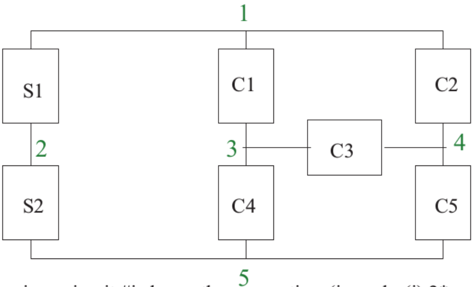
Motivation



PF coils power supply circuits, [Havlicek,2011]

Circuit matrix



S : supply

C : coil

1-5 : circuit node

For the given circuit #i above, the connections(i,nnodes(i),2*ncomponent) matrix reads:

	S1(1)	S1(2)	S2(1)	S2(2)	C1(1)	C1(2)	C2(1)	C2(2)	C3(1)	C3(2)	C4(1)	C4(2)	C5(1)	C5(2)
1[[	1	0	0	0	1	0	1	0	0	0	0	0	0	0]
2 [	0	1	1	0	0	0	0	0	0	0	0	0	0	0]
3 [	0	0	0	0	0	1	0	0	1	0	1	0	0	0]
4 [	0	0	0	0	0	0	0	1	0	1	0	0	1	0]
5 [	0	0	0	1	0	0	0	0	0	0	0	1	0	1]]

Matrix describing general circuit

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Equipotential

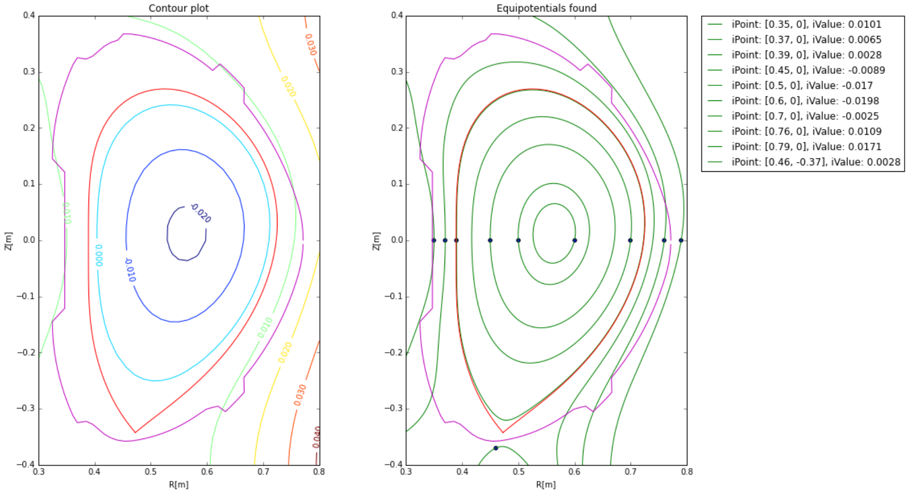
- Additional dependency $x(\lambda)$, $y(\lambda)$

$$\begin{aligned} F(x,y) &= 0, \\ \frac{\partial F}{\partial x}\frac{dx}{d\lambda} + \frac{\partial F}{\partial y}\frac{dy}{d\lambda} &= 0, \\ (d\lambda)^2 &= (dx)^2 + (dy)^2, \end{aligned}$$

- results in set of differential equations

$$\begin{aligned} \frac{dx}{d\lambda} &= \mp\frac{F_y}{\sqrt{F_x^2 + F_y^2}},\ x(0) = x_0, \\ \frac{dy}{d\lambda} &= \pm\frac{F_x}{\sqrt{F_x^2 + F_y^2}},\ y(0) = y_0. \end{aligned}$$

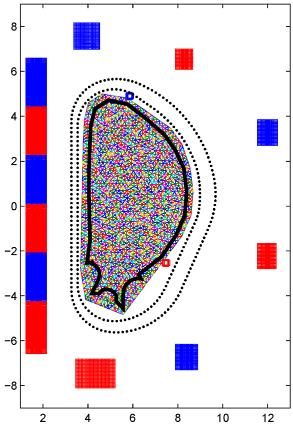
My own contour plot



Comparison of contour plot and algorithm results

Problem and possible solution

- Data grid
- SmoothBivariateSpline class smoothing factor
- My own spline class



Triangulation grid used by FREEBIE code,[Urban,2012]

References

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- URBAN, J. and J.-F. ARTAUD. *Coupled transport and free-boundary equilibrium simulations using FREEBIE and CRONOS*. 2012.

The End

Bonus: Green's function

Definition

- $LG(x,s) \; = \; \delta(x-s)$

Motivation

- $Lu(x) \; = \; f(x)$
- $\int LG(x,s)f(s)ds \; = \; \int \delta(x-s)f(s)ds \; = \; f(x)$
- $u(x) = \int G(x,s)f(s)ds$