Monte Carlo Simulation of the Two Stream Instability

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Research project

Building a Computer Model

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Lagged Fibonacci Generators (LFG)

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Inverse Matrix

$$\boldsymbol{I} = \begin{pmatrix} \cos\theta\cos\varphi & -\sin\varphi & \sin\theta\cos\varphi \\ \cos\theta\sin\varphi & \cos\varphi & \sin\theta\sin\varphi \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \tag{6}$$

Random Angles Generation

$(0, 0, u)^t$ Rotation by Φ and Θ

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$$\delta = \tan\Theta/2 \tag{9}$$

$$\sigma^2 = \frac{e_\alpha^2 e_\beta^2 n_L \lambda}{8\pi\epsilon_0^2 m_{\alpha\beta}^2 u^3}\Delta t \tag{10}$$

Results of the Binary Collision

$$\begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}^{t+\Delta t} = \boldsymbol{I} \begin{pmatrix} u \sin \Theta \cos \Phi \\ u \sin \Theta \sin \Phi \\ u \cos \Theta \end{pmatrix} \tag{11}$$

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$$\boldsymbol{v}_\alpha^{t+\Delta t} = \boldsymbol{v}_\alpha^t + \frac{m_{\alpha\beta}}{m_\alpha}\Delta\boldsymbol{u} \tag{13}$$

$$\boldsymbol{v}_\beta^{t+\Delta t} = \boldsymbol{v}_\beta^t - \frac{m_{\alpha\beta}}{m_\beta}\Delta\boldsymbol{u} \tag{14}$$

Remarks

$$\lambda_{1,2} = 23 - \ln \left[\frac{Z_1 Z_2 (\mu_1 + \mu_2)}{\mu_1 T_2 + \mu_2 T_1} \left(\frac{n_1 Z_1^2}{T_1} + \frac{n_2 Z_2^2}{T_2} \right)^{1/2} \right] \quad (15)$$

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$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \quad (16)$$

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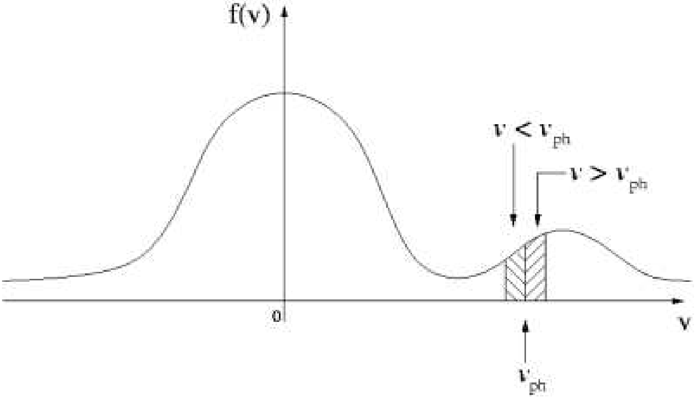
$$\sigma = \sqrt{\frac{k_\mathrm{B} T}{m}} \tag{18}$$

$$\langle v^2\rangle - \langle v\rangle^2 = \frac{k_\mathrm{B} T}{m}\left(3 - \frac{8}{\pi}\right) \tag{19}$$

Two Stream Instability



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- Collisions in separated Maxwellians neglected, straightforward motion, temperature calculation

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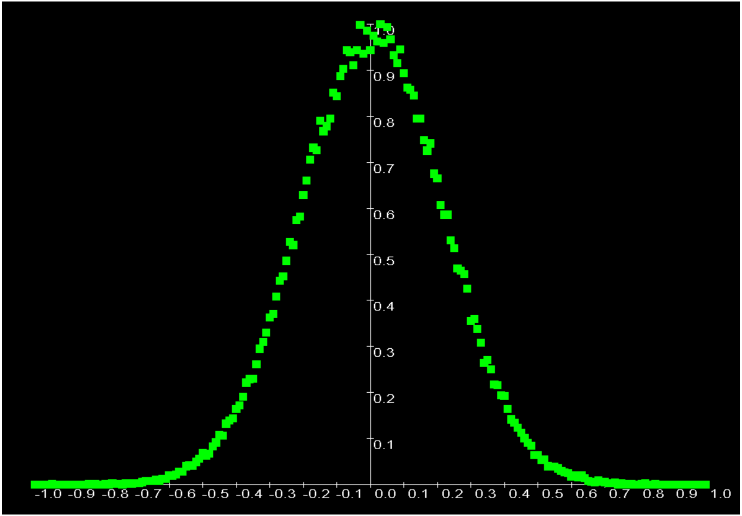
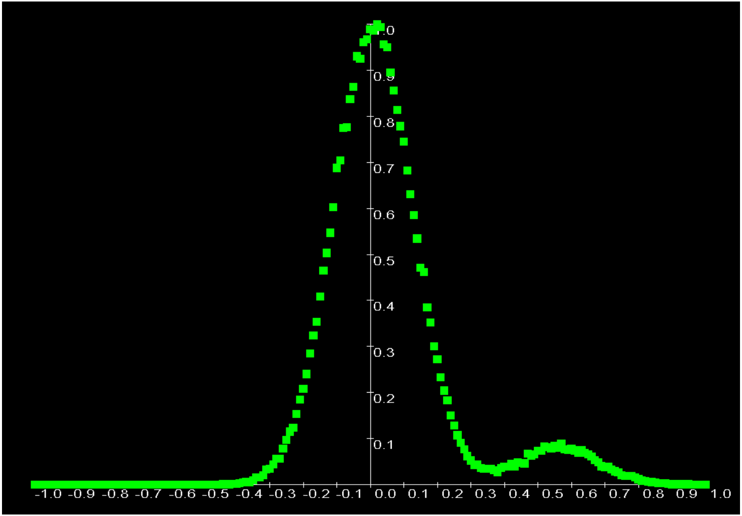
- The beam heated up to 3 keV, the target to 2 keV

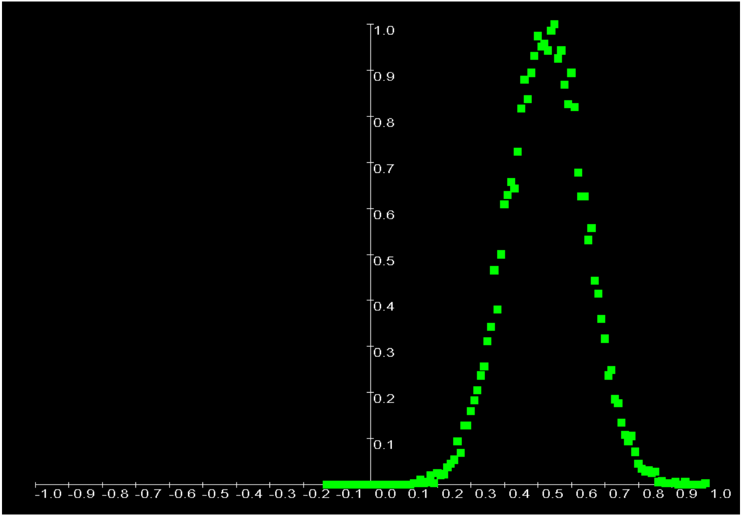
Initial target velocities distribution



Final target velocities distribution



Initial beam velocities distribution



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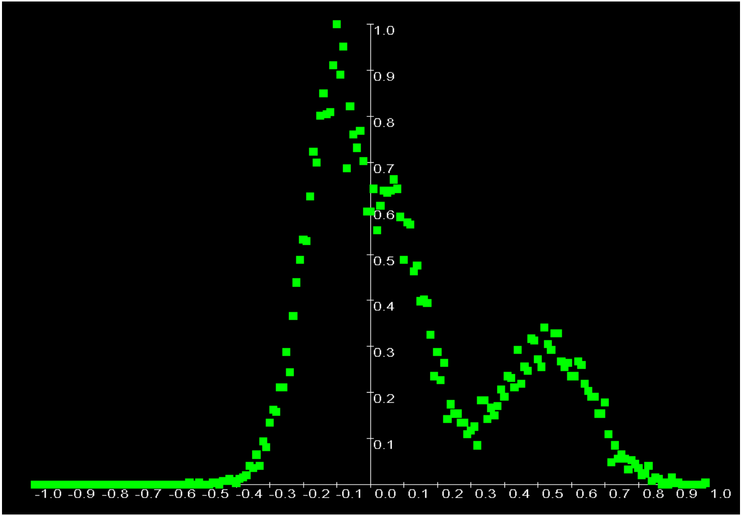

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