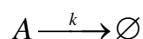# Euler's method in Excel to simulate simple differential equation models

It is shown how to implement Euler's method in Excel to approximately solve an initial-value problem (IVP). Excel 2007 was used.

As example we take a model of an irreversible molecular decay reaction:
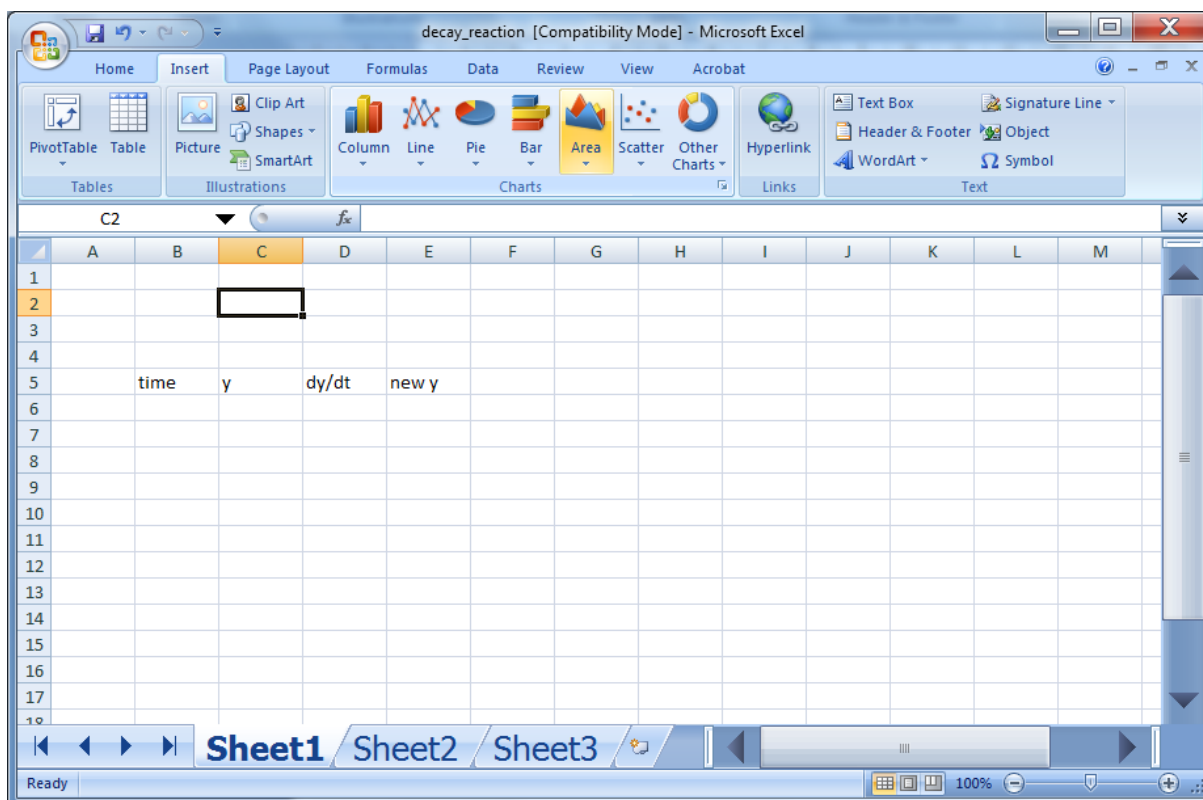
$$A \xrightarrow{\ k\ } \varnothing$$

Which, according to the law of mass action, can be described with the following (linear, first order) differential equation:
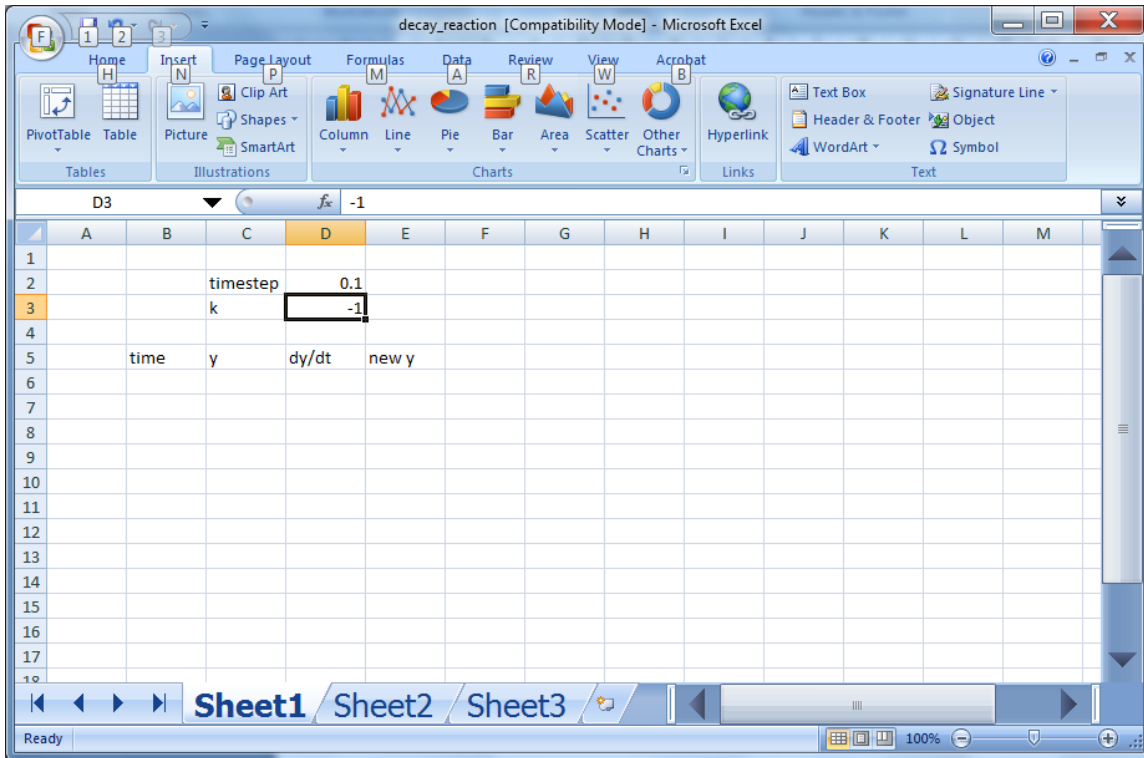
$$\frac{dy}{dt} = -ky \qquad \text{with } y = [A]$$
$$y(0) = 5$$

We begin by creating four column headings, labelled as shown, in our Excel spreadsheet. (Skip a few lines at the top of the spreadsheet, as in the example below.)

Somewhere above your table headings, label a cell "timestep", and enter the timestep size you want to use in the cell next to it. In the same way, introduce the parameter k, like this:

Next, enter the values of the initial time (0) and initial condition (5) in the first two columns, as shown:
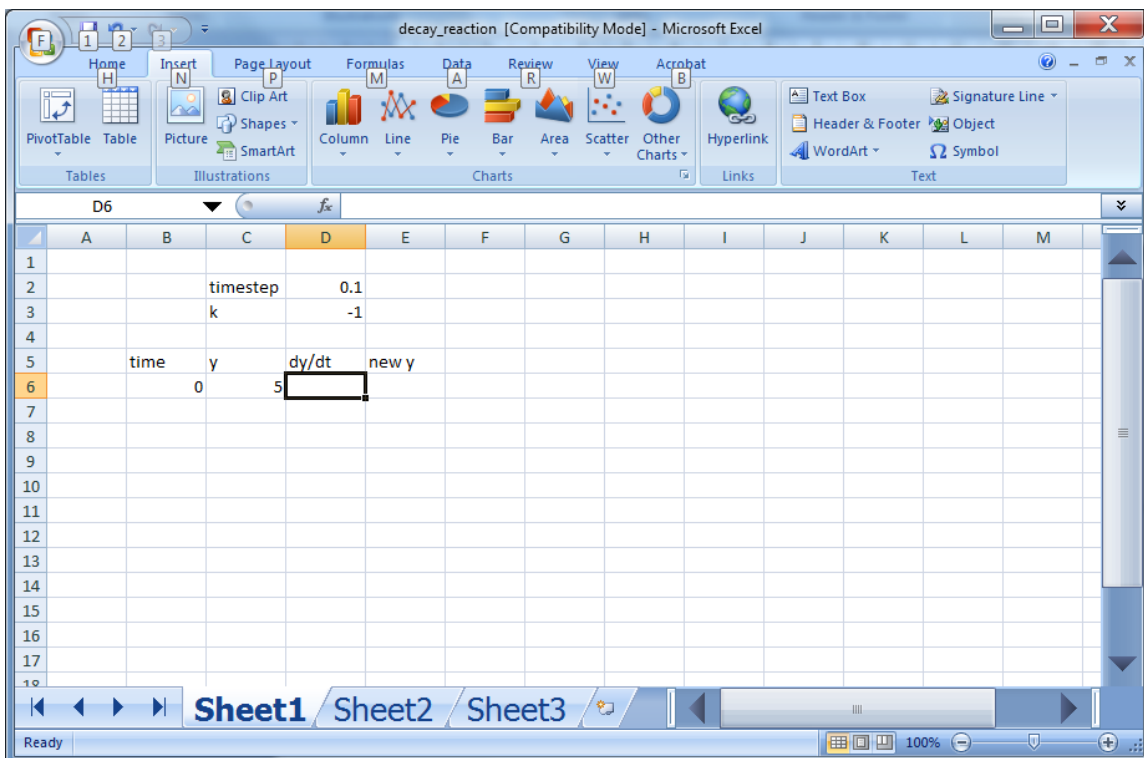
Next, we will complete the column of values of *time*, by using the fact that each time value is the previous *time* value plus the *timestep*. So, to compute the second *time*-value in the time column, we enter the following formula into cell B7:

=B6+$D$2

Note the use of the $ in the cell address for the timestep. This results in the value 0.1 appearing in cell B6.

Now, highlight and drag cell B6 down the column, until you reach the desired ending value. (*Pull down the small black square that appears in bottom-right corner of a cell if you select a cell*.) This will repeat the operation in B7 in all the lower cells. Because of the $ in the cell address for the timestep, the same timestep is used in all cells.

For example, if we choose to stop Euler's method at *time*=1, our spreadsheet would look like this:
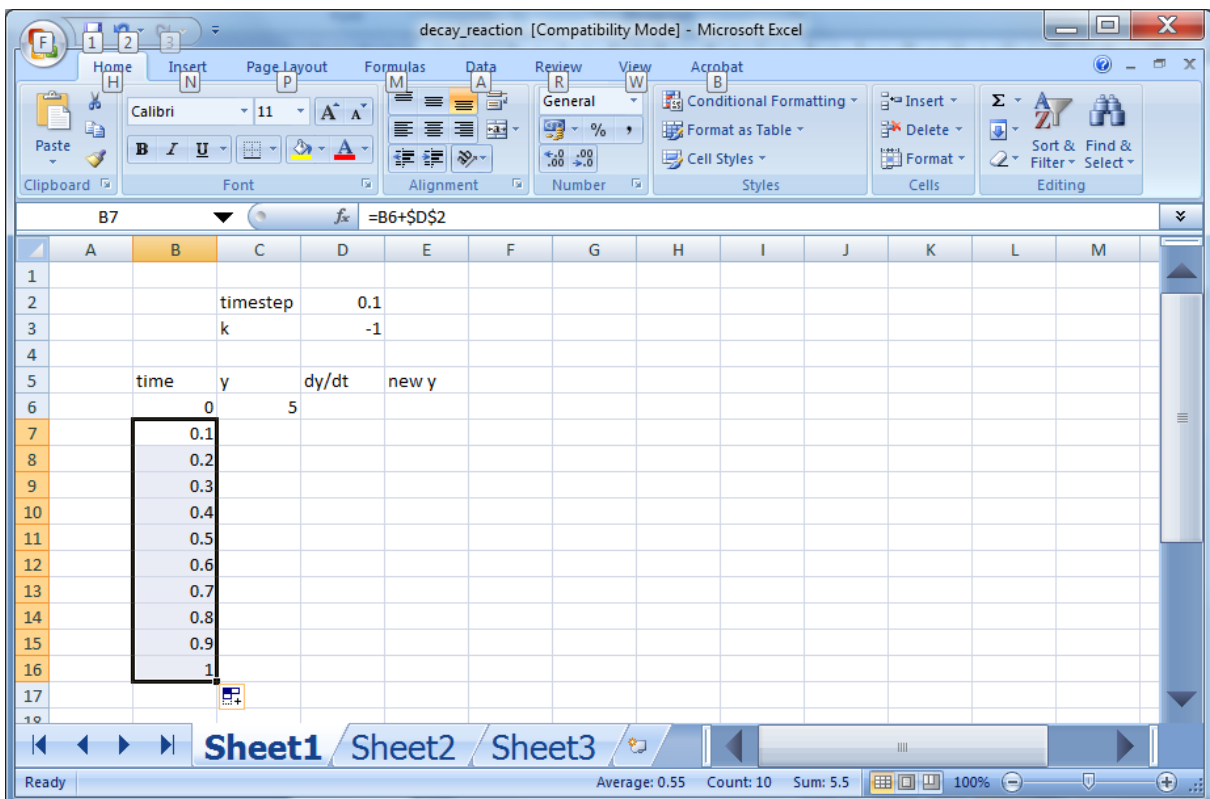
We use the right-hand side of the differential equation to compute the value for the first cell in the dy/dt column. In D6 we enter:

=$D$3*C6

where we have substituted cell addresses in place of parameter *k* and variable *y*. We get:

The Euler's method formula

$$\text{new } y = y + dy/dt*\text{timestep}$$

becomes the Excel formula (to be entered in cell E6):
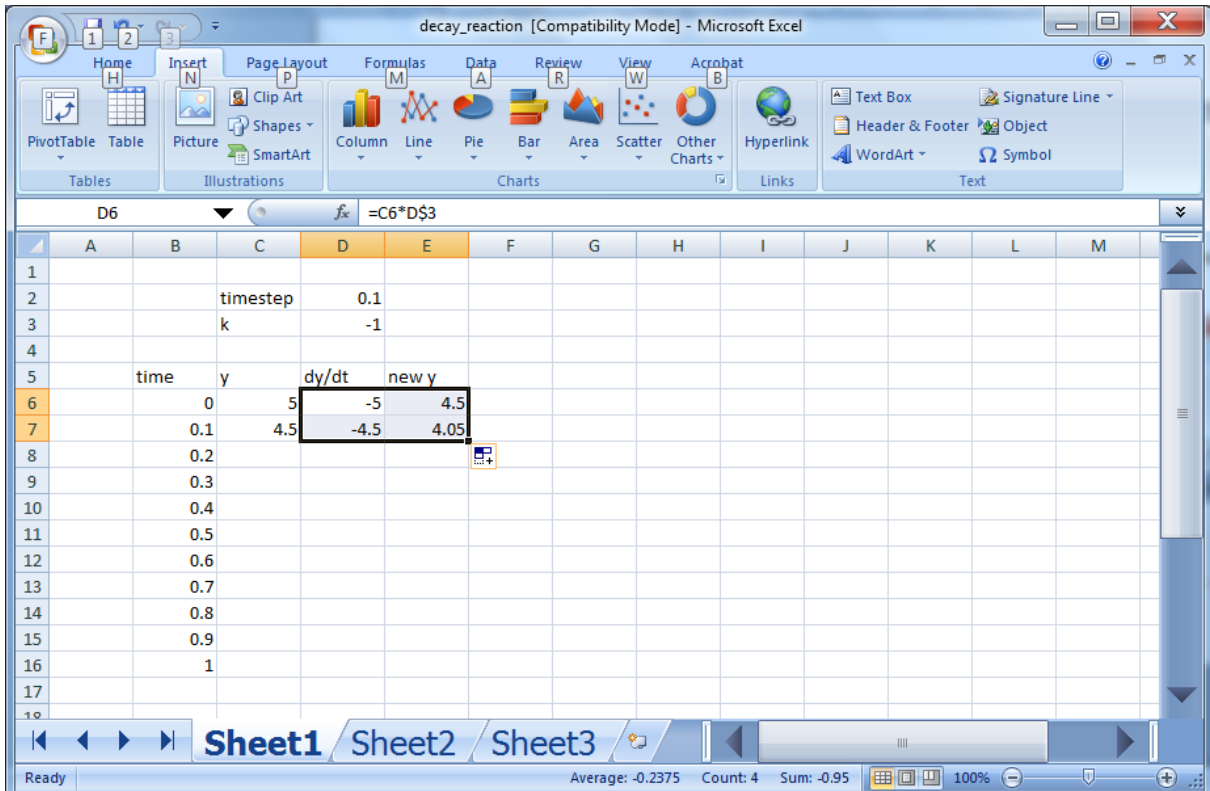
$$=C6+D6*\$D\$2$$

Our spreadsheet now looks like this:

Next, we enter the next y-value in the table. This is simply the first entry in the "new y" column. Hence, we enter the following into cell C7:
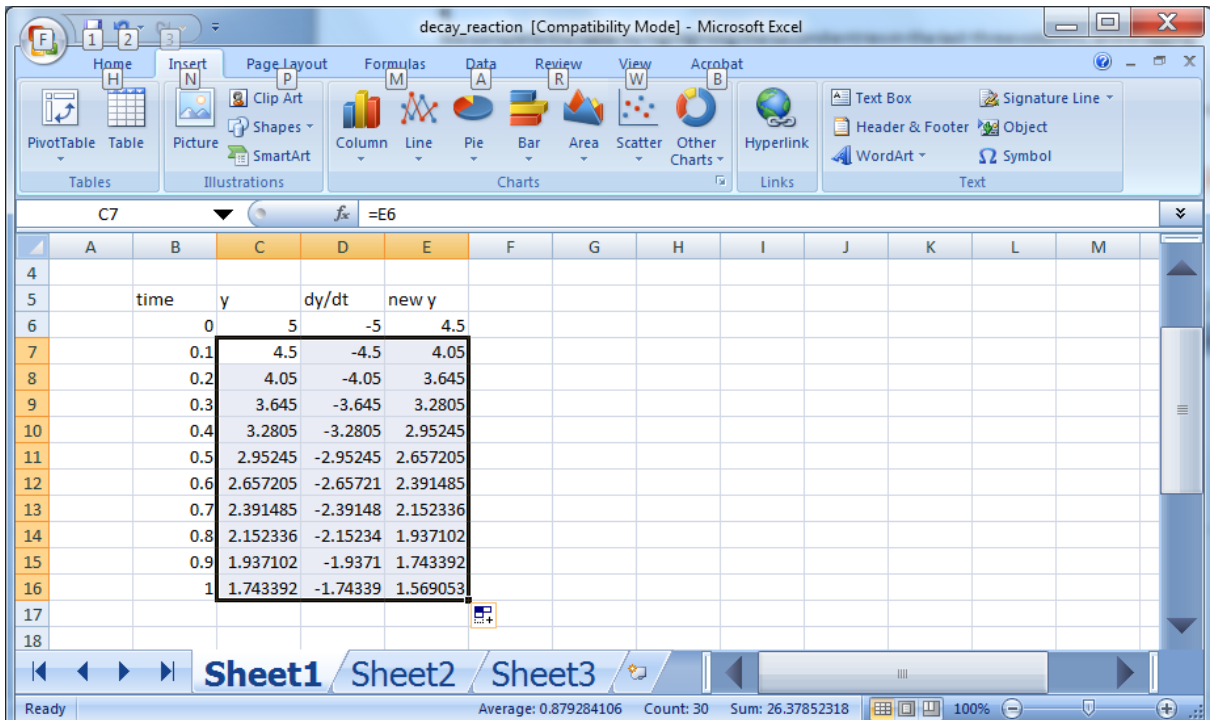
$$=E6$$

This moves the value 4.5 into the y column.

Next, highlight the first cells in the last two columns, and drag them down one row to complete the second column of the table.

We complete the table by highlighting the second entries in the last three columns, and dragging them down as far as the time-column goes:

To extend the simulation for a longer time period, highlight the bottom row of the current table and drag it down as far as you want:

	time	y	dy/dt	new y
	0	5	-5	4.5
	0.1	4.5	-4.5	4.05
	0.2	4.05	-4.05	3.645
	0.3	3.645	-3.645	3.2805
	0.4	3.2805	-3.2805	2.95245
	0.5	2.95245	-2.95245	2.657205
	0.6	2.657205	-2.65721	2.391485
	0.7	2.391485	-2.39148	2.152336
	0.8	2.152336	-2.15234	1.937102
	0.9	1.937102	-1.9371	1.743392
	1	1.743392	-1.74339	1.569053
	1.1	1.569053	-1.56905	1.412148
	1.2	1.412148	-1.41215	1.270933
	1.3	1.270933	-1.27093	1.14384
	1.4	1.14384	-1.14384	1.029456
	1.5	1.029456	-1.02946	0.92651
	1.6	0.92651	-0.92651	0.833859
	1.7	0.833859	-0.83386	0.750473
	1.8	0.750473	-0.75047	0.675426
	1.9	0.675426	-0.67543	0.607883

To create a plot of y versus time, select the first two columns, from the Insert tab, select a <u>Scatter</u> plot (choose one that will plot a line). The data in the first column will be automatically plotted on the x-axis and the data in the second column will be on the y-axis.
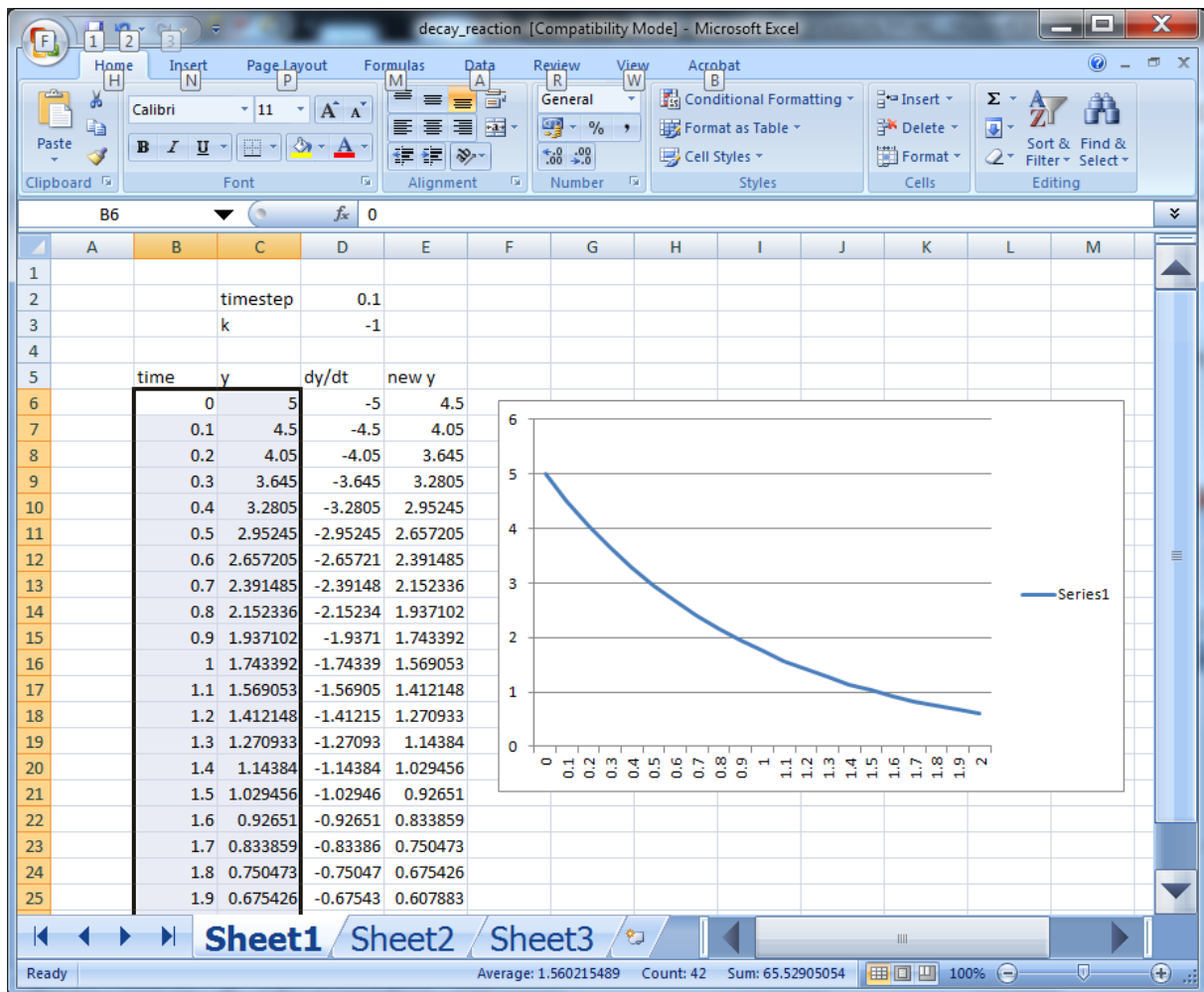
Exercises

1. Change the initial condition from 5 to $y(0) = 2$. Observe that the simulation is updated automatically
2. Increase the rate constant k to 10 and run the simulation with a timestep of 1. Subsequently, change the timestep such that an accurate simulation result is obtained.