

Plasma diagnostics
in basic plasma physics devices and tokamaks:
from principles to practice

Theory of electrostatic probes

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Content of unit

- **Langmuir probe (LP) examples**
- **Theory of single LP**
 - Analysis of the sheath
 - Fluid plasma description
 - The I-V characteristic
 - How to extract plasma parameters
- **Langmuir probes in practice**
 - How to implement a LP
 - Typical experimental problems and how to face them
 - Sheath expansion, bandwidth limitation, probe contamination
 - Langmuir probes in magnetic fields
- **More complex electrostatic probes**
 - Double LP, Triple LP, harmonic method for T_e
 - Katsumata, ballpen probes
- **Electrostatic analyzers**
 - Druyvesteyn method for $f(v)$ measurements, grid energy analyzer

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The Langmuir probe

One of the simplest and certainly one of the most used plasma diagnostics. Usually associated with Irving Langmuir who was one of the first to use electric probes to directly sense plasma fluxes.

Good for low temperature plasmas: $T_e \leq 100 \text{ eV}$

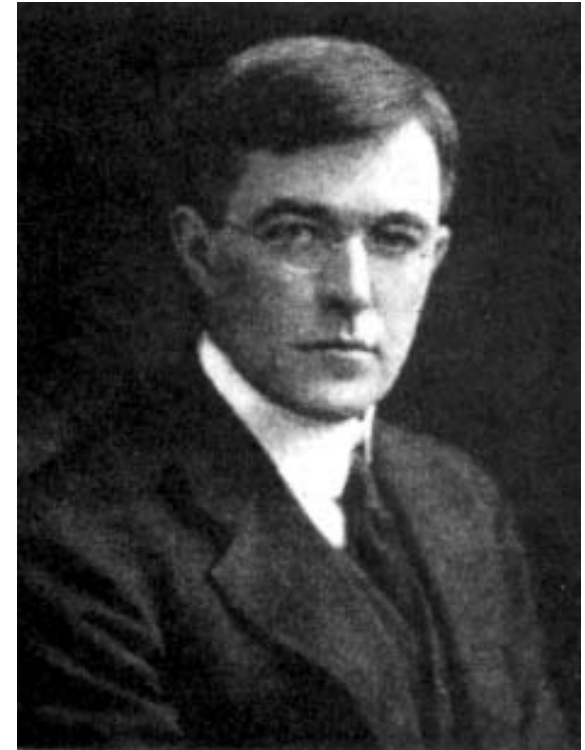
Particularly well adapted for typical SOL/divertor conditions. Widely used in basic plasma physics devices.

- $1 \leq T_e \leq 100 \text{ eV}$, $10^{12} \leq n_e \leq 10^{20} \text{ m}^{-3}$

Probes are generally quite robust and cheap – can be embedded into limiters and divertor tiles or inserted quickly into the SOL using fast reciprocating drives.

In basic plasma physics devices, small energy flux to the probe \rightarrow no need to reciprocate ($T_e < 50 \text{ eV}$, $n_e < 10^{18}$, $t < 2 \text{ s}$).

Disadvantage is that “proper” interpretation of probe data can be notoriously difficult.



Irving Langmuir

31/01/1881-16/08/1957

Nobel prize for
chemistry in 1932.

Originator of the term
“plasma”

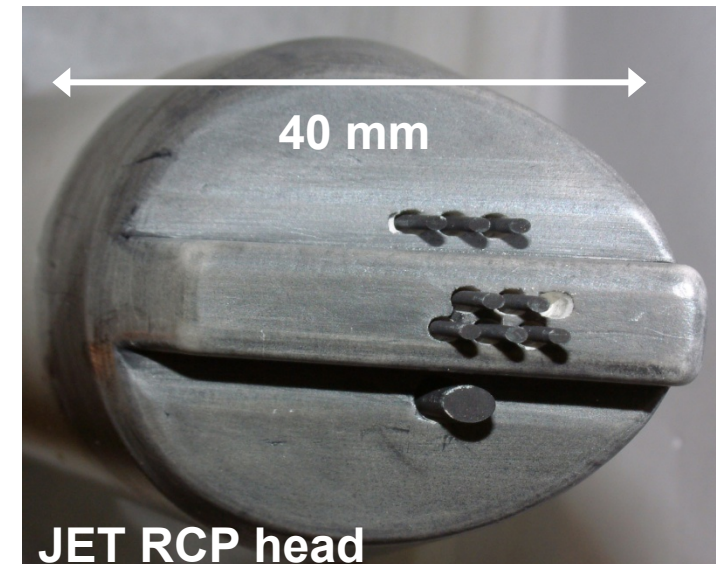
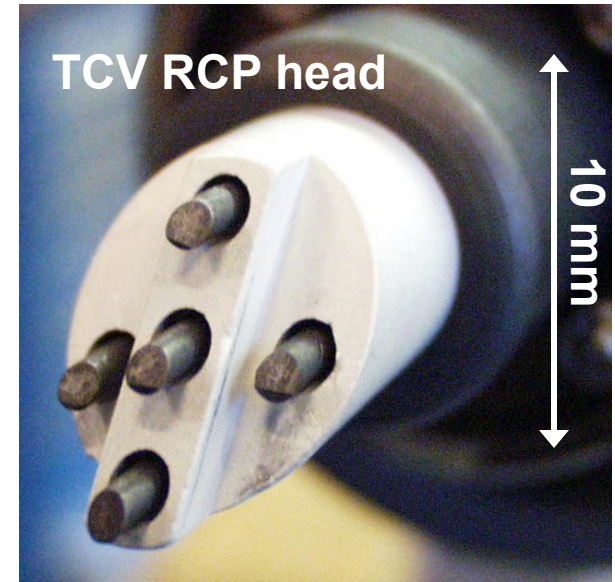
LPs in fusion devices

Probes can come in **single, double and triple** probe varieties and an enormous range of designs and shapes. **Single probe is the most common.**

In most tokamaks, where power handling is a serious issue there are now **2 distinct types** of probe design:

Fast reciprocating probes (RCP) - for SOL plasma profile measurements and fluctuation studies. Probe movements are of order 100 ms and probe tips usually cylindrical with “normal” incidence to the magnetic field.

Wall-mounted - in tokamaks dominated by carbon (e.g.. TCV, DIII-D, JT-60U), probe tips usually machined in graphite. Typically, the pins will be embedded in a low Z, refractory material (often Boron nitride – since many tokamaks are also “boronised”). This provides electrical insulation, allowing bias voltages to be applied to the pins.



Wall-mounted LPs in TCV

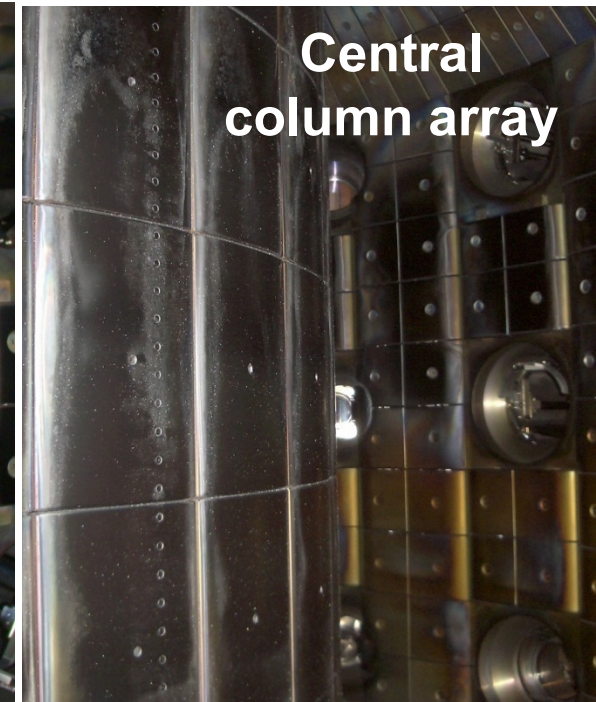
Divertor or limiter embedded probes – deduce parameters at the plasma-surface interface.

Many probes required in tile arrays to generate a useful profile.

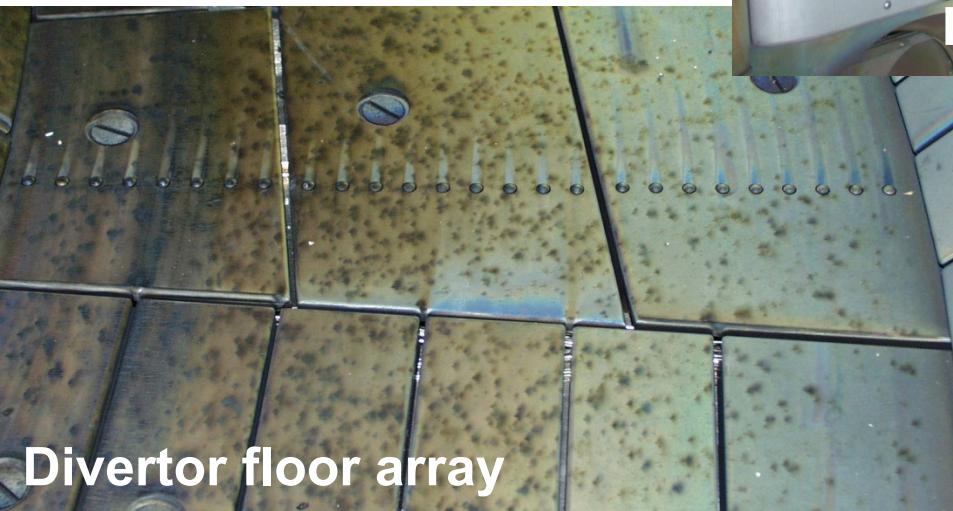
Advantage is that probe “looks” like the tile and thus does not perturb the plasma.



LFS wall array



Central column array



Divertor floor array

TCV Probes are of the “**domed**” or “**button**” design (LFS, outer divertor) and “flush mounted” (central column).

Domed only possible in areas where heat fluxes not too high or prolonged.

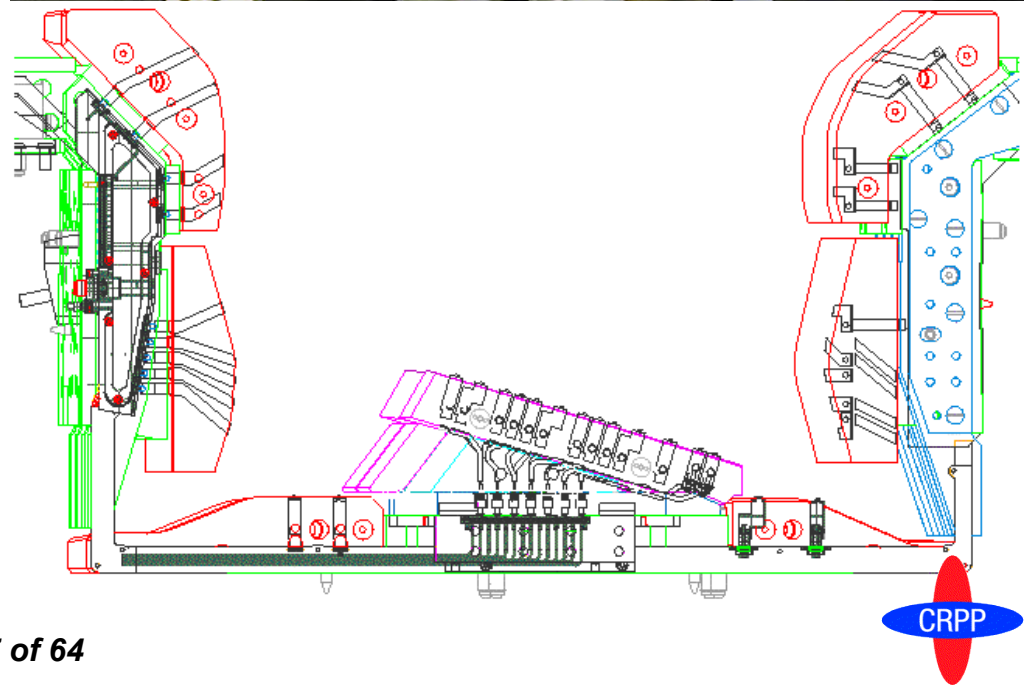
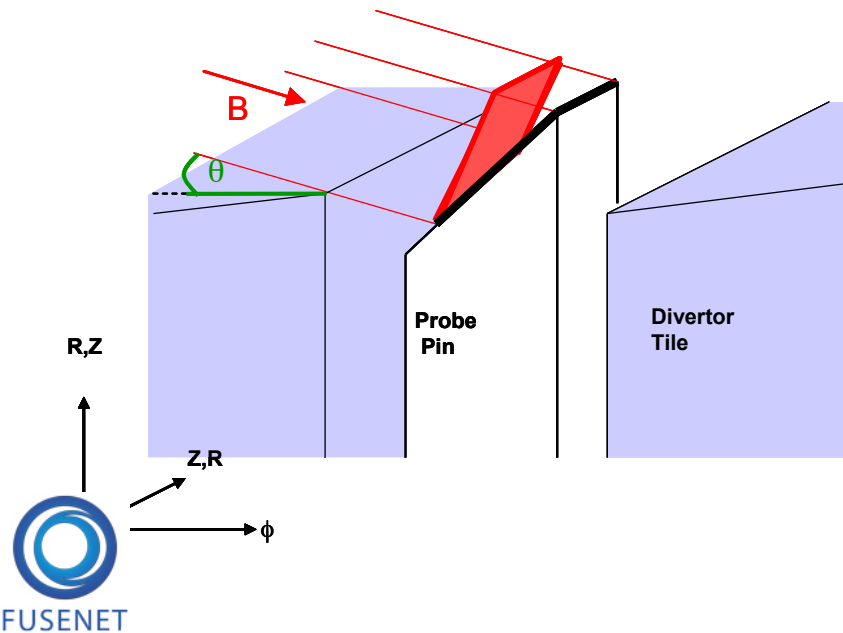
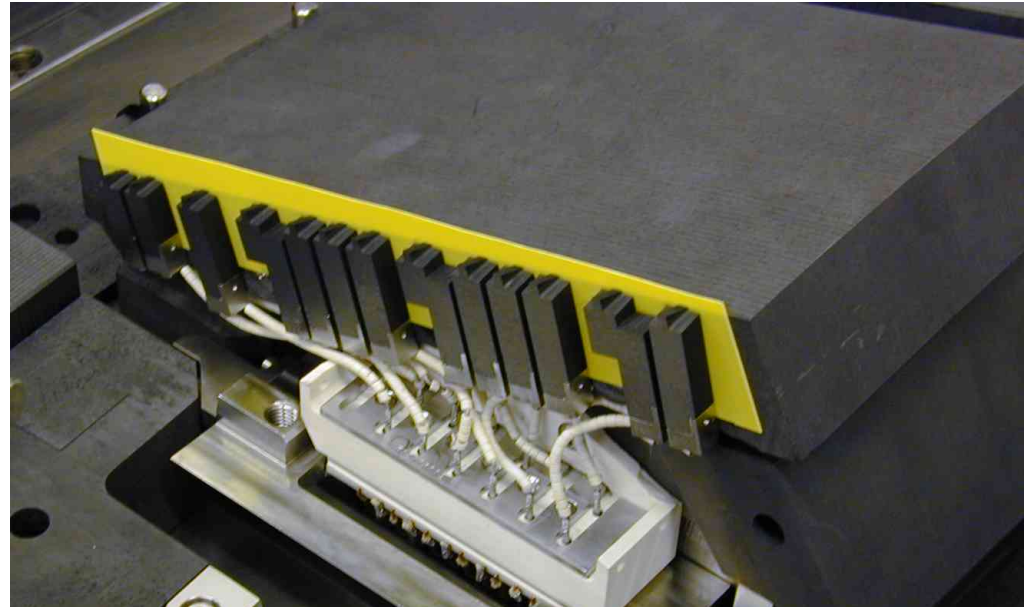
Flush mounted can be difficult at low field line angles (as in the divertor).

Different LP concept used at JET

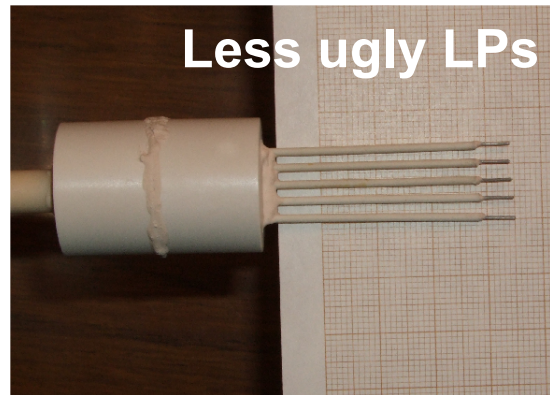
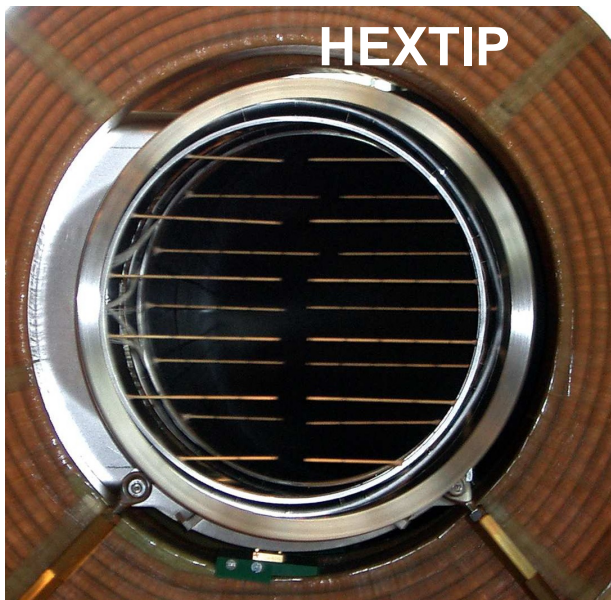
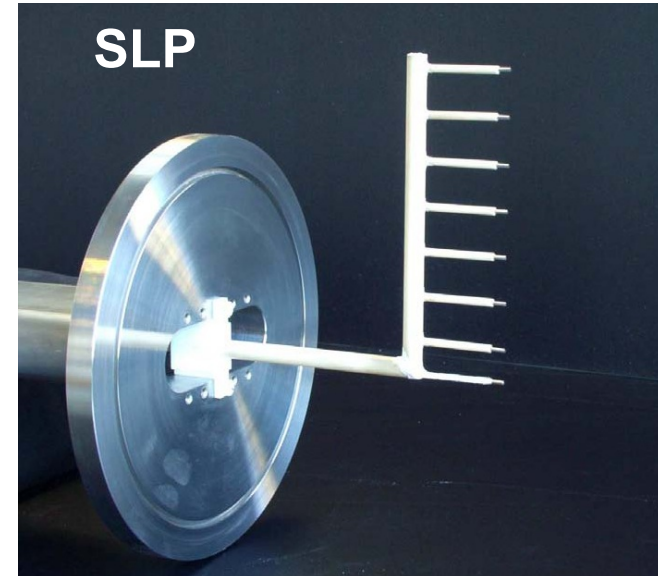
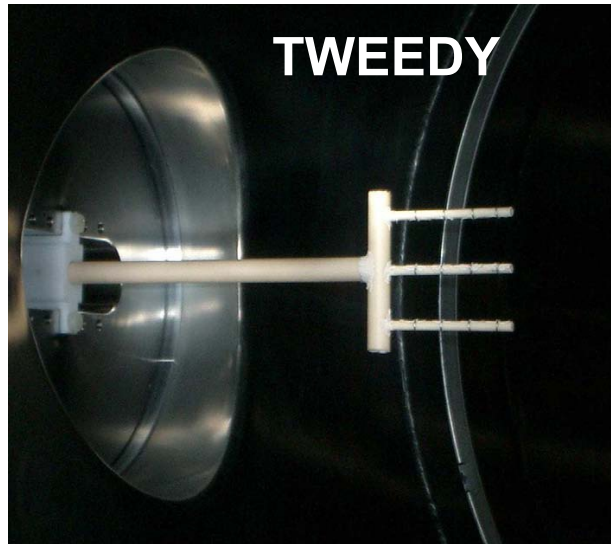
Arrays of probes arranged in between divertor tile modules with well defined “rectangular” geometry.

Three separate neighboring toroidal sectors equipped nearly identically to provide “**triple probe**” capability for fast T_e measurements (e.g. during ELMs).

Probes made in strong CFC for durability – conditions very harsh in the JET divertor.



LPs in basic plasma physics devices

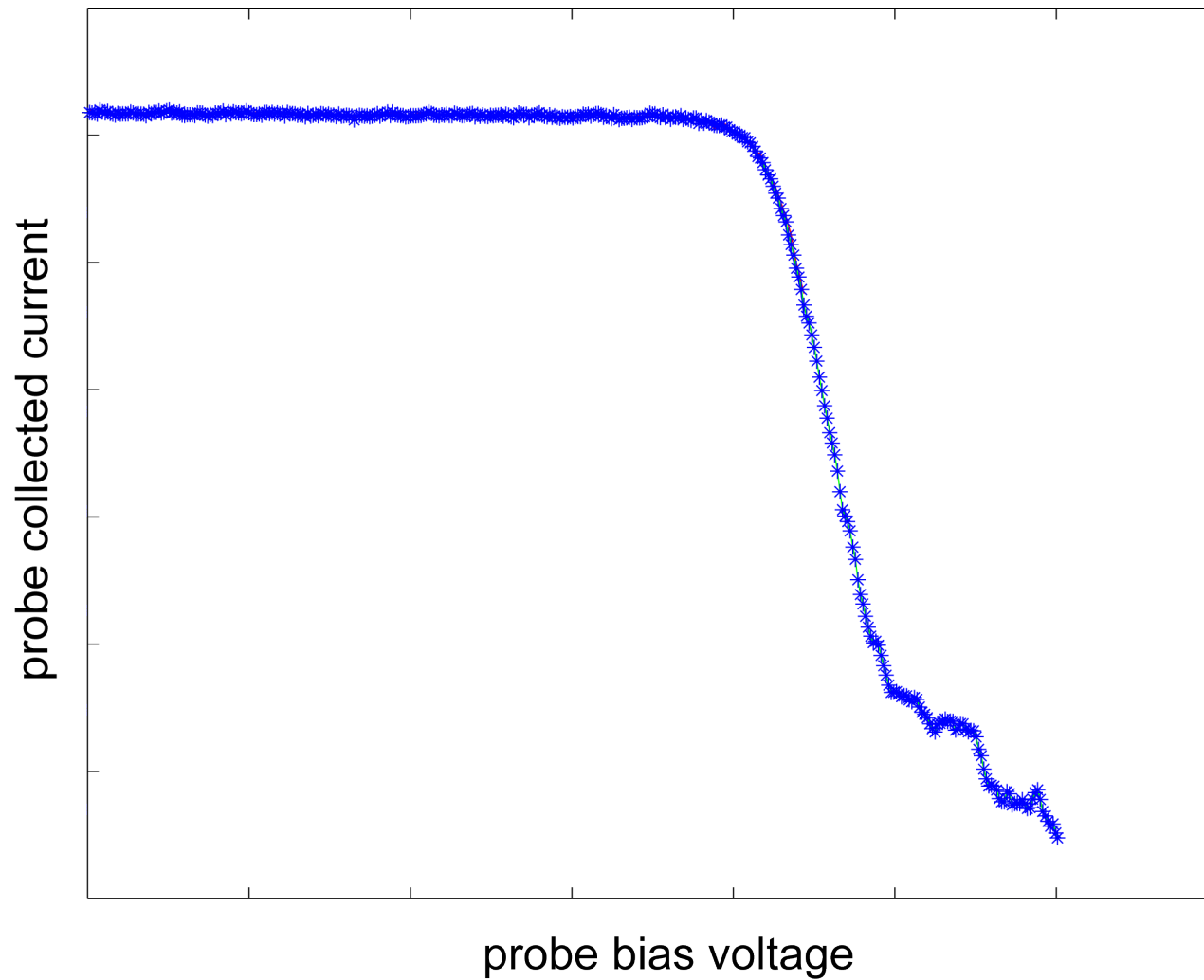


In basic plasma physics devices there is no need to reciprocate.

LPs can be placed directly inside the plasma.

Tips are usually made of tungsten, stainless steel or molybdenum.

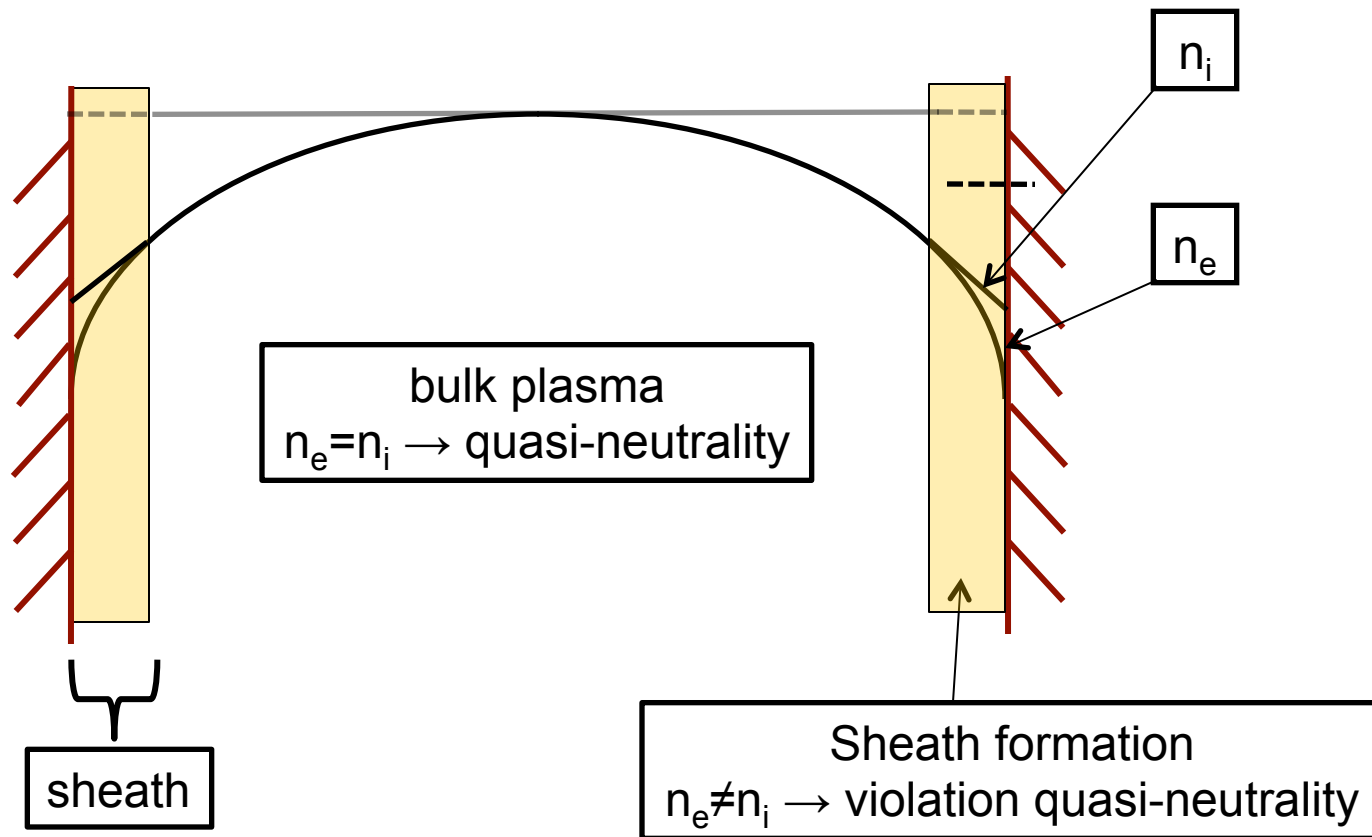
Current-Voltage (I-V) LP characteristics



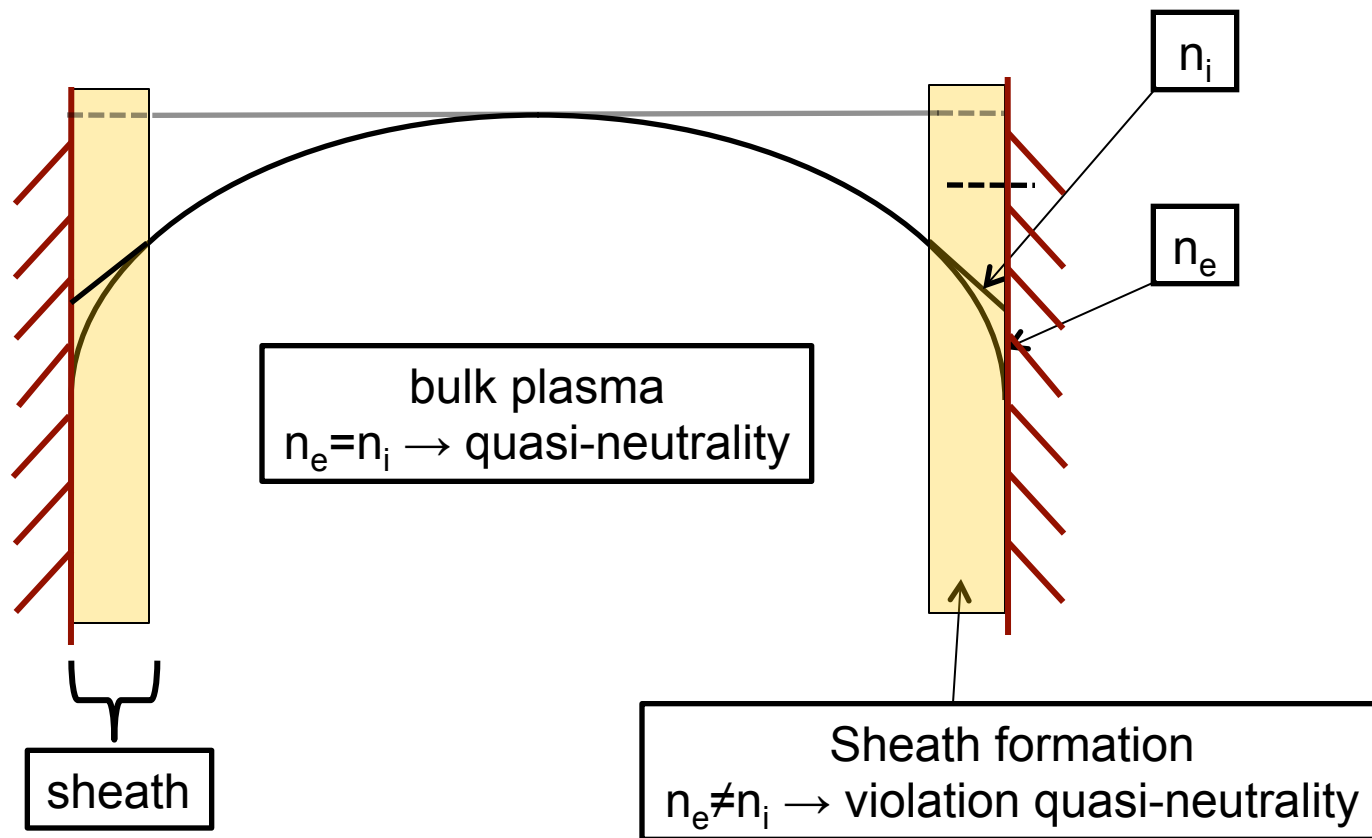
Interaction of plasmas with material surfaces

bulk plasma
 $n_e = n_i \rightarrow$ quasi-neutrality

Interaction of plasmas with material surfaces

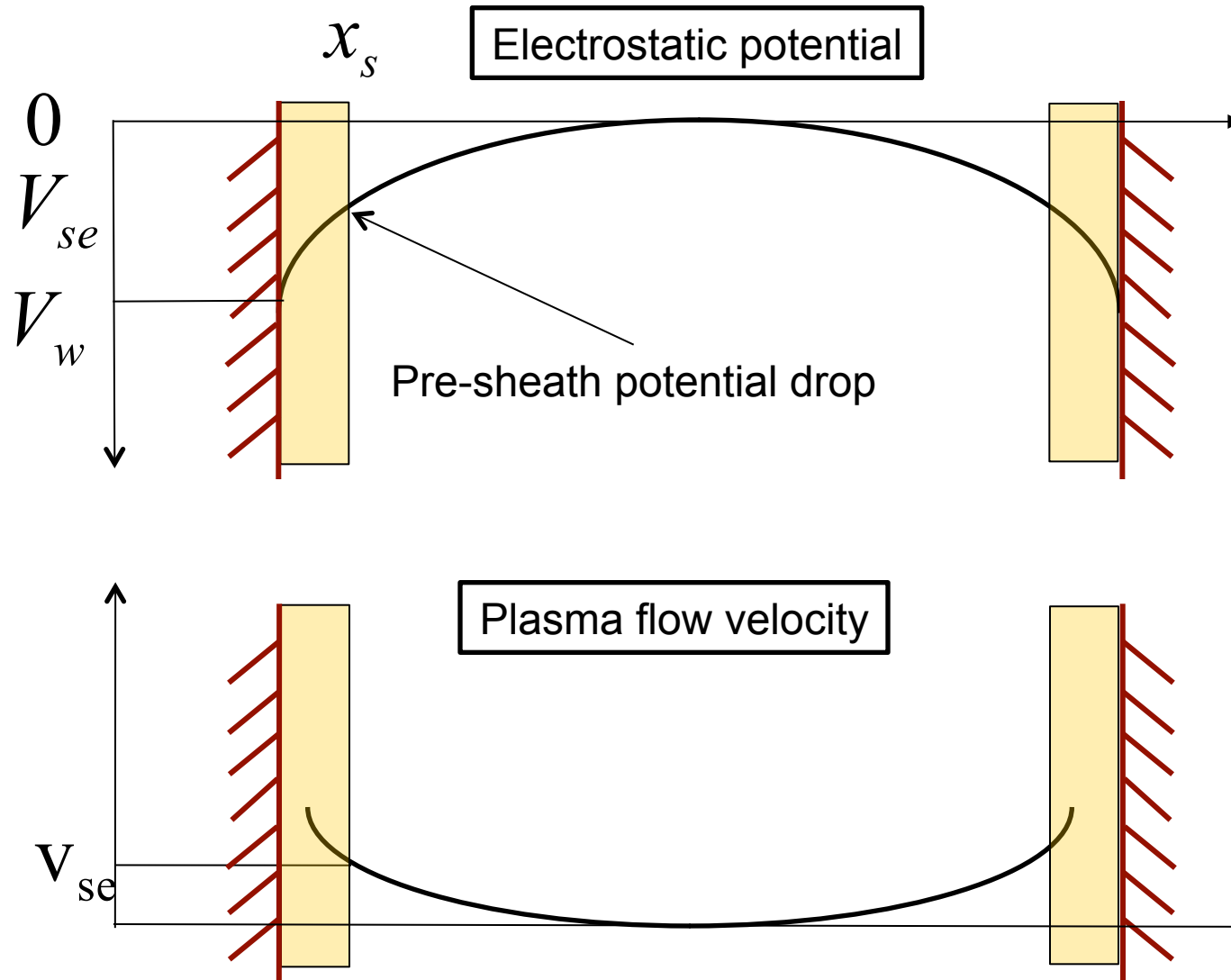


Interaction of plasmas with material surfaces



The goal of any probe model is to determine the unperturbed values (in the absence of the probe) of plasma parameters from the measured values of current and voltage at the probe

Analysis of the sheath



Hypothesis

Ion point source
 $T_i = 0$
 zero velocity
 Ions are collisionless

Goal

Determine the ion velocity at the sheath edge using the conservation of energy

$$\frac{1}{2} m_i V_{se}^2 = -eV_{se}$$

Analysis of the sheath: the ions

$$\left. \begin{array}{ll} \frac{1}{2} m_i v^2 = -eV & \text{Conservation of energy} \\ n_i v = \text{const} & \text{Particle conservation} \end{array} \right\} n_i = n_{se} (V_{se}/V)^{1/2}$$

In the plasma, we could use the quasi-neutrality conditions but not in the sheath.
In the sheath, we can use the Maxwell equation:

$$\frac{d^2 V}{dx^2} = -\frac{e}{\epsilon_0} (n_i - n_e)$$

$$\nabla \cdot E = \frac{e}{\epsilon_0} (n_i - n_e)$$

1D Poisson Equation

Analysis of the sheath: the electrons

$$n_e(x) = n_{se} \exp[e(V - V_{se}) / kT_e]$$

$$n_i = n_{se} (V_{se} / V)^{1/2}$$

We assume that the electrons are Maxwellian in the sheath and that the electron temperature remain constant. Therefore the electron density falls off according to a Boltzman factor.

Using these two equations into the 1D Poisson equation we obtain:

$$\frac{d^2 V(x)}{dx^2} = -\frac{e}{\epsilon_0} n_{se} \left[\left(\frac{V_{se}}{V(x)} \right)^{1/2} - \exp\{e[V(x) - V_{se}] / kT_e\} \right]$$

Let us focus on a thin region inside the sheath

$$\Delta = V_{se} - V > 0 \quad \left\{ \begin{array}{l} \left(\frac{V_{se}}{V} \right)^{1/2} \approx 1 + \frac{1}{2} \frac{\Delta}{V_{se}} = 1 - \frac{1}{2} \frac{\Delta}{|V_{se}|} \\ \exp[e(V - V_{se}) / kT_e] \approx 1 - \frac{e\Delta}{kT_e} \end{array} \right.$$

$$\frac{d^2 \Delta}{dx^2} = \frac{e\Delta}{\epsilon_0} n_{se} \left[\frac{e}{kT_e} - \frac{1}{2|V_{se}|} \right]$$

$$y'' = \alpha y \quad \alpha > 0 \quad \text{Exponential solution}$$

$$\alpha < 0 \quad \text{Oscillatory solution} \rightarrow \text{unphysical and not seen experimentally}$$

The *exit velocity* of the ions: Bohm's criterion

Exponential solution

$$\alpha > 0$$

$$\frac{e}{kT_e} \geq \frac{1}{2|V_{se}|} \quad \rightarrow \quad m_i V_{se}^2 \geq kT_e$$

$$V_{se} \leq -T_e / 2e$$

Bohm's criterion for the ion exit velocity

$$V_{se} \geq c_s = \sqrt{\frac{kT_e}{m_i}}$$

No sheath forms for $V_{se} > -T_e/2e$. Probes near the plasma potential need not be surrounded by a sheath. The plasma is therefore quasi-neutral up to the probe surface.

Rough estimate of the sheath thickness

$$\frac{d^2 \Delta}{dx^2} = \frac{e\Delta}{\epsilon_0} n_{se} \left[\frac{e}{kT_e} - \frac{1}{2|V_{se}|} \right]$$

$$\frac{\Delta}{L_{sheath}^2} \approx \frac{e\Delta}{\epsilon_0} n_{se} \frac{e}{kT_e}$$

$$L_{sheath} \approx \sqrt{\frac{\epsilon_0 kT_e}{e^2 n_{se}}} \equiv \lambda_{Debye}$$

TCV

$$n_e = 10^{19} \text{m}^{-3}$$

$$T_e = 20 \text{eV}$$

$$\lambda_{Debye} \sim 0.01 \text{ mm}$$

TORPEX

$$n_e = 10^{16} \text{m}^{-3}$$

$$T_e = 5 \text{eV}$$

$$\lambda_{Debye} \sim 0.1 \text{ mm}$$

More refined analysis show that the sheath is usually 15-30 Debye length thick.

Fluid description of the plasma

$$\frac{d\Gamma}{dx} = \frac{d(nv)}{dx} = S_p$$

Particle conservation

$$F = ma = m \frac{dv}{dt} = m \frac{dv}{dx} \frac{dx}{dt} = mv \frac{dv}{dx}$$

Equation of motion

$$nF = neE - \frac{dp}{dx} - nF_{drag} = neE - \frac{dp}{dx} - mvS_p$$

It is the momentum required per m³ per s to bring the ions instantly up to the local speed

$$nm \, v \frac{dv}{dx} = neE - \frac{dp}{dx} - mvS_p$$

Momentum equation

The electron fluid

$$nm_e v \frac{dv}{dx} = -neE - \frac{dp_e}{dx} - m_e v S_p$$

Momentum conservation

$$nm_e v \frac{v}{L} = -neE - \frac{kn_e T_e}{L} - m_e v \frac{nv}{L}$$

Using the particle conservation and recalling that a typical velocity is c_s .

$$nm_e \frac{c_s^2}{L} = -neE - \frac{kn_e T_e}{L} - m_e \frac{nc_s^2}{L}$$

Momentum equation

~~$$n \frac{m_e}{m_i} \frac{2T_e}{L} = -neE - \frac{kn_e T_e}{L} - n \frac{m_e}{m_i} \frac{2T_e}{L}$$~~

$$\rightarrow neE = \frac{dp}{dx}$$

The force balance for electrons is between the electric field and the pressure gradients → Boltzmann factor

Ions + electrons = plasma fluid

Hypothesis

Singly charged ions

$$n_e = n_i$$

Ambipolar flow

Ions and electrons
are produced at the
same rate

Isothermal plasma

$$\frac{d\Gamma}{dx} = \frac{d(nv)}{dx} = S_p \quad \text{Particle conservation}$$

Momentum equation for ions

$$nm_i v \frac{dv}{dx} = -neE - \frac{dp_i}{dx} - m_i v S_p$$

Boltzmann equation for electrons

$$n_e e E = -kT_e \frac{dn_e}{dx}$$

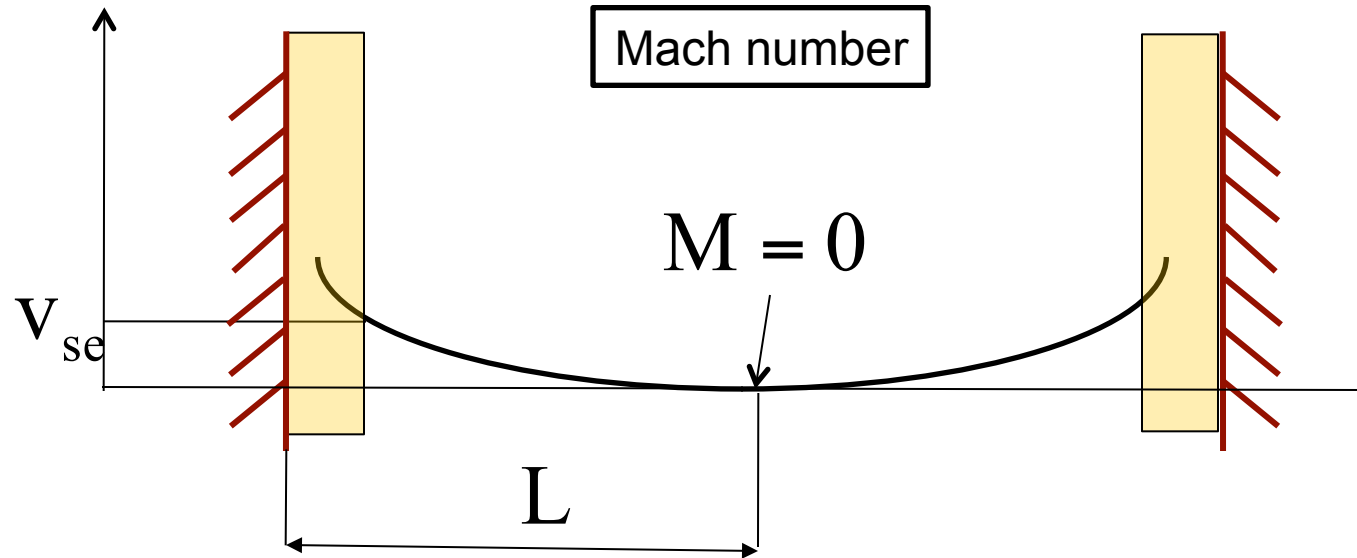
Using the particle conservation and defining the Mach number: $M \equiv v / c_s$

$$nm v \frac{dv}{dx} = -mc_s^2 \frac{dn}{dx} - m v S_p$$

The flow is sonic at the sheath edge

$$\frac{dM}{dx} = \frac{S_p}{nc_s} \frac{(1+M^2)}{(1-M^2)}$$

$$\frac{dM}{dx} > 0$$



We note that M cannot exceed unity (supersonic flow) without introducing an unphysical singularity in the bulk plasma. However we found from analysis on the sheath side that at the sheath entrance the velocity must be larger than c_s . **We conclude therefore that at the sheath edge the flow is sonic, i.e.**

$$V_{se} = c_s$$

Density variation in the bulk: pre-sheath density drop

1D fluid, no source, by combining ion and electron momentum equation

$$\frac{d}{dx}(p_e + p_i + nm v^2) = 0$$

Conservative form of the momentum equation

$$p_e + p_i + nm v^2 = \text{const}$$

$$n(x) = \frac{n_0}{(1 + M^2)}$$

$$n(L) = n_{se} = \frac{n_0}{2}$$

For isothermal condition the pre-sheath density drop is only a factor of 2.

Voltage variation in the bulk: pre-sheath voltage drop

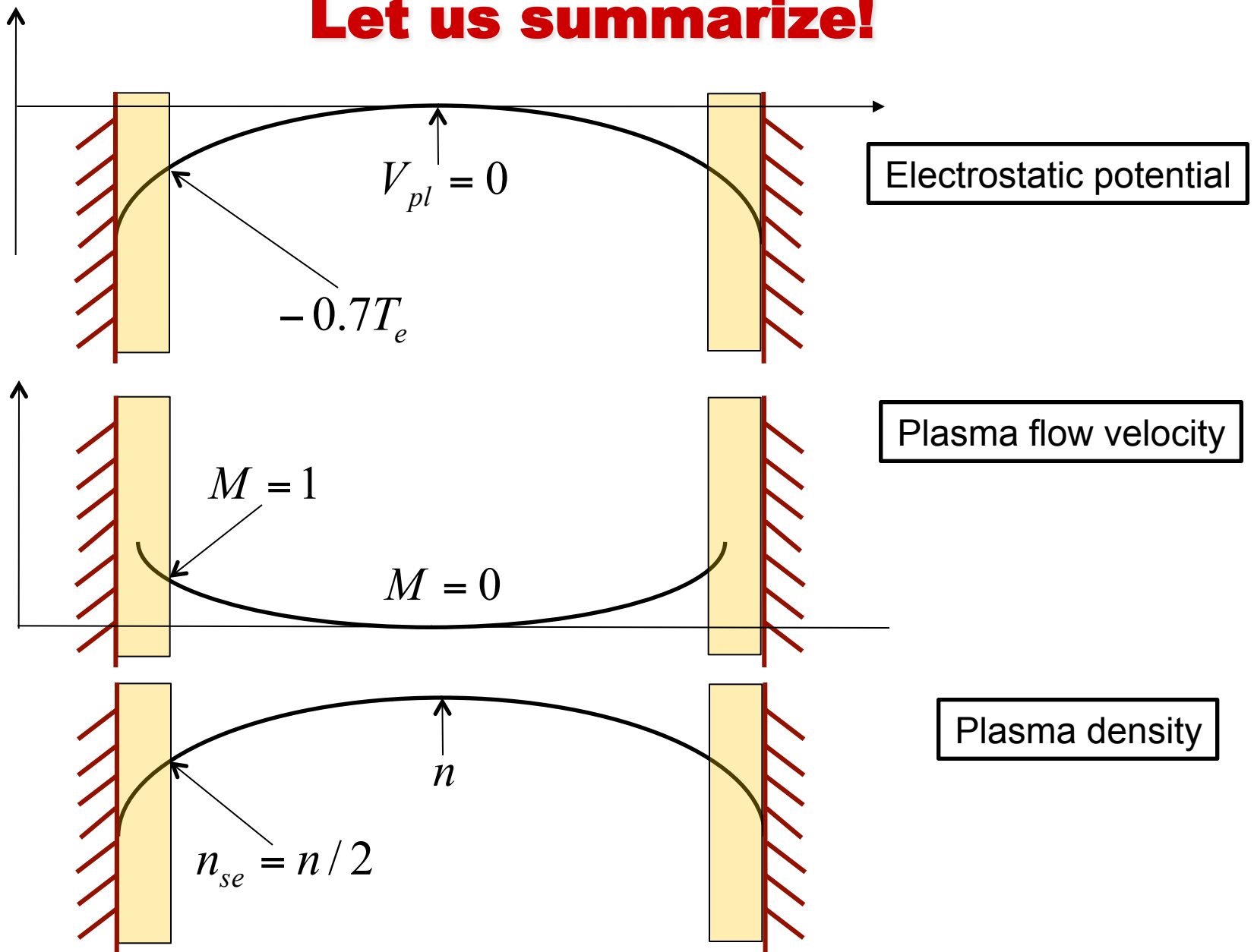
$$n(x) = \frac{n}{(1 + M^2)}$$

$$n(x) = n \exp(eV / kT_e)$$

$$V(x) = -kT_e \log[1 + M(x)^2]$$

$$V(L) = V_{se} = -kT_e \log(1/2) \approx -0.7kT_e$$

Let us summarize!



Maxwellian distribution: a few properties

$$f^{Max}(v_x, v_y, v_z) = c \exp \left\{ -\frac{b}{2} m \left[(v_x - a_x)^2 + (v_y - a_y)^2 + (v_z - a_z)^2 \right] \right\}$$

$$w = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$f^{Max}(w) = 4\pi w^2 \exp \left\{ -\frac{b}{2} m w^2 \right\}$$

$$b = -1 / kT$$

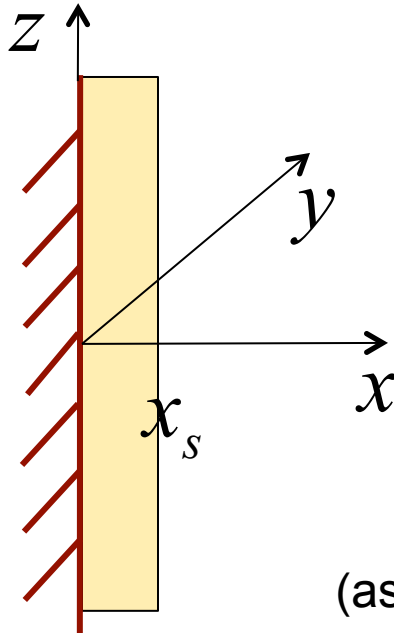
$$c = n (m / 2\pi kT)^{3/2}$$

$$\beta = m / 2kT$$

$$f^{Max}(w) = n (\beta / \pi)^{3/2} \exp \{ -\beta w^2 \}$$

$$\langle w \rangle = \frac{1}{n} \int_0^\infty f^{Max}(w) w dw = 2 (2kT / \pi m)^{1/2} = \bar{c}$$

Electron and ion flux at the wall



$$\delta n = f(v) dv_x dv_y dv_z$$

$$\delta \Gamma_x = v_x \delta n = v_x f(v) dv_x dv_y dv_z$$

The electrons are Maxwellian and therefore the unidirectional flux can be estimated as
(assuming that the plasma potential is zero at the sheath entrance):

$$\Gamma_x = \int_0^{\infty} v_x f(v) dv_x \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z = \frac{1}{4} n \bar{c}$$

$$\Gamma_i = n_{wi} c = n_{se} c_s$$

$$\Gamma_e = \frac{1}{4} n_w \bar{c}_e = \frac{1}{4} n_{se} \bar{c}_e \exp(eV_w / kT_e)$$

What happens is the surface is floating?

If the surface is floating, no current can flow. We have then ambipolar flow: $\Gamma_i = \Gamma_e$
This condition defines the floating potential, i.e. the potential that naturally arises between the plasma and a surface electrically isolated.

$$\Gamma_i = n_{wi} c = n_{se} c_s \qquad \Gamma_e = \frac{1}{4} n_w \bar{c}_e = \frac{1}{4} n_{se} \bar{c}_e \exp(eV_w / kT_e)$$

$$\frac{eV_{fl}}{kT_e} = \log\left(\frac{I_{e,sat}}{I_{i,sat}}\right) = \log \sqrt{\left[\left(2\pi \frac{m_e}{m_i} \right) \right]} \quad \Lambda = \log \sqrt{\left[\left(2\pi \frac{m_e}{m_i} \right) \left(1 + \frac{T_i}{T_e} \right) \right]}$$

$$V_{fl} = V_{pl} - \Lambda T_e$$

For Hydrogen, typically $\Lambda \sim 3$. However it should be determined experimentally, since the electron saturation current may be reduced in the presence of a magnetic field.

The total current drawn by a surface - I

$$\Gamma_e = \frac{1}{4} n_w \bar{c}_e = \frac{1}{4} n_{se} \bar{c}_e \exp(eV_w / kT_e) \quad \text{Electron flux}$$

$$\Gamma_i = n_{se} c_s = \frac{1}{4} n_{se} \bar{c}_e \exp(eV_{fl} / kT_e) \quad \text{Ion flux recalling the expression for the floating potential}$$

The total current drawn by the probe is computed as the sum of the ion and electron currents:

$$j = e(\Gamma_i - \Gamma_e) = \frac{1}{4} n_{se} \bar{c}_e \left[\exp(eV_{fl} / kT_e) - \exp(eV_w / kT_e) \right]$$

The total current drawn by a surface - II

$$j = \frac{1}{4} en_{se} \bar{c}_e \exp(eV_{fl} / kT_e) \{1 - \exp[(eV_w - V_{fl}) / kT_e]\} =$$

$$= en_{se} c_s \{1 - \exp[e(V_w - V_{fl}) / kT_e]\}$$

The final formula is obtained by recalling the pre-sheath density drop formula $n_{se} = n/2$

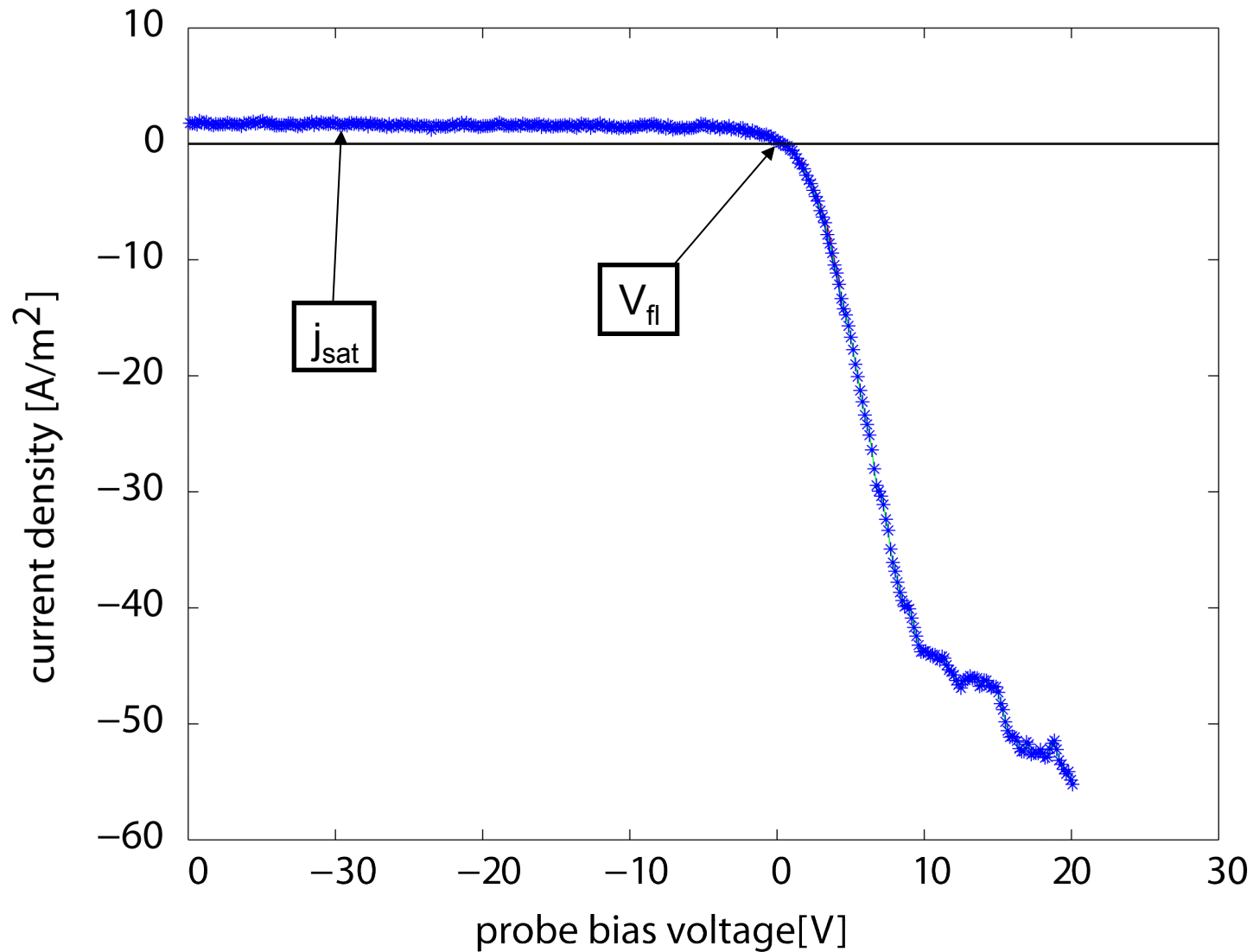
$$j = e \frac{n}{2} c_s \{1 - \exp[e(V_w - V_{fl}) / kT_e]\}$$

Ion saturation
current density

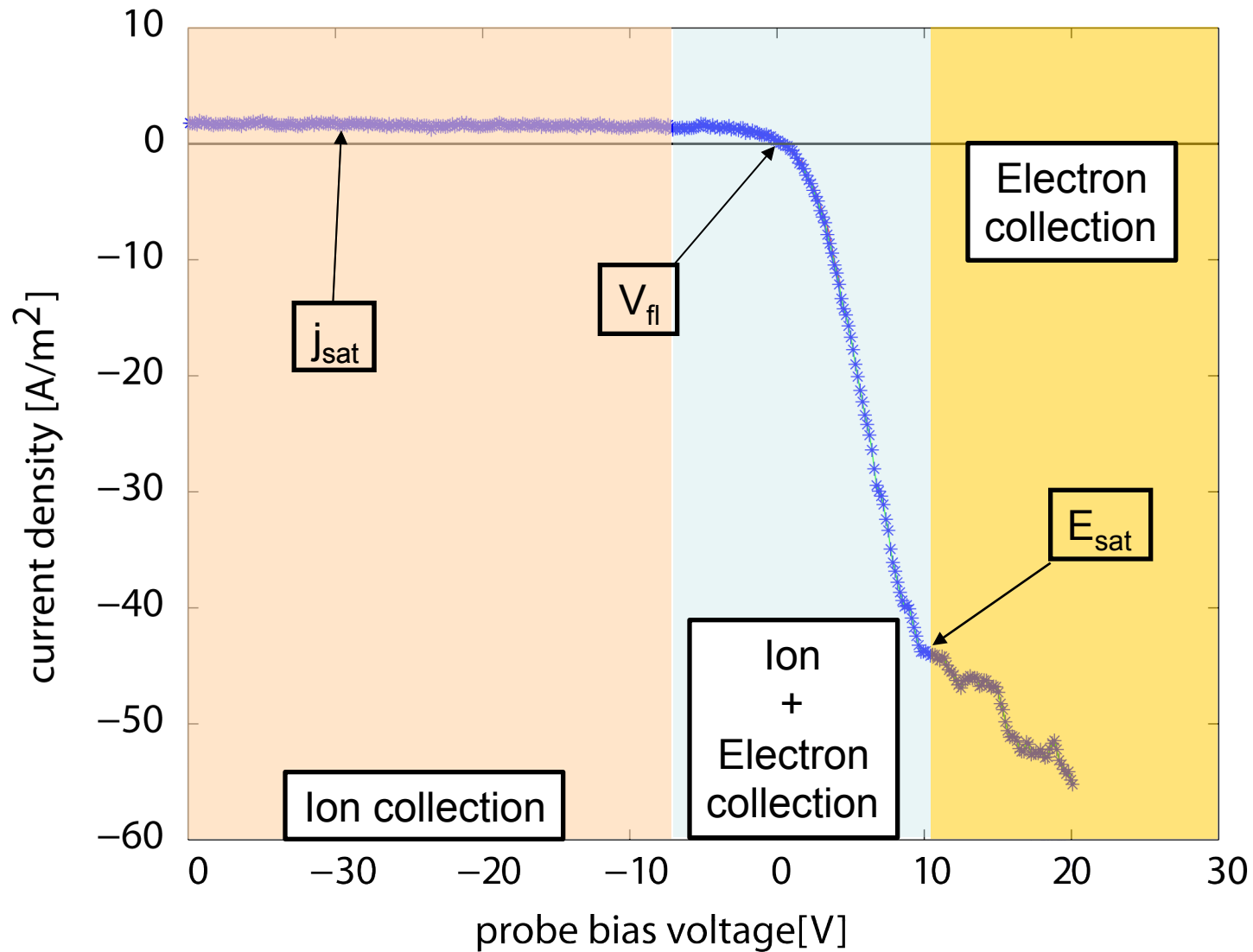
$$j = j_{sat} \{1 - \exp[e(V_w - V_{fl}) / kT_e]\}$$

$$I = A_p j_{sat} \{1 - \exp[e(V_w - V_{fl}) / kT_e]\}$$

$$I_{pr} = I_{sat} [1 - \exp(V_{pr} - V_f)/T_e]$$



$$I_{pr} = I_{sat} [1 - \exp(V_{pr} - V_f)/T_e]$$



How to extract plasma parameters

The knee of the I-V characteristics identifies the plasma potential V_{pl} .

V_f : the “floating potential” \rightarrow no current driven to the probe.

Plasma potential: $V_{pl} = V_{fl} + \Delta T_e$

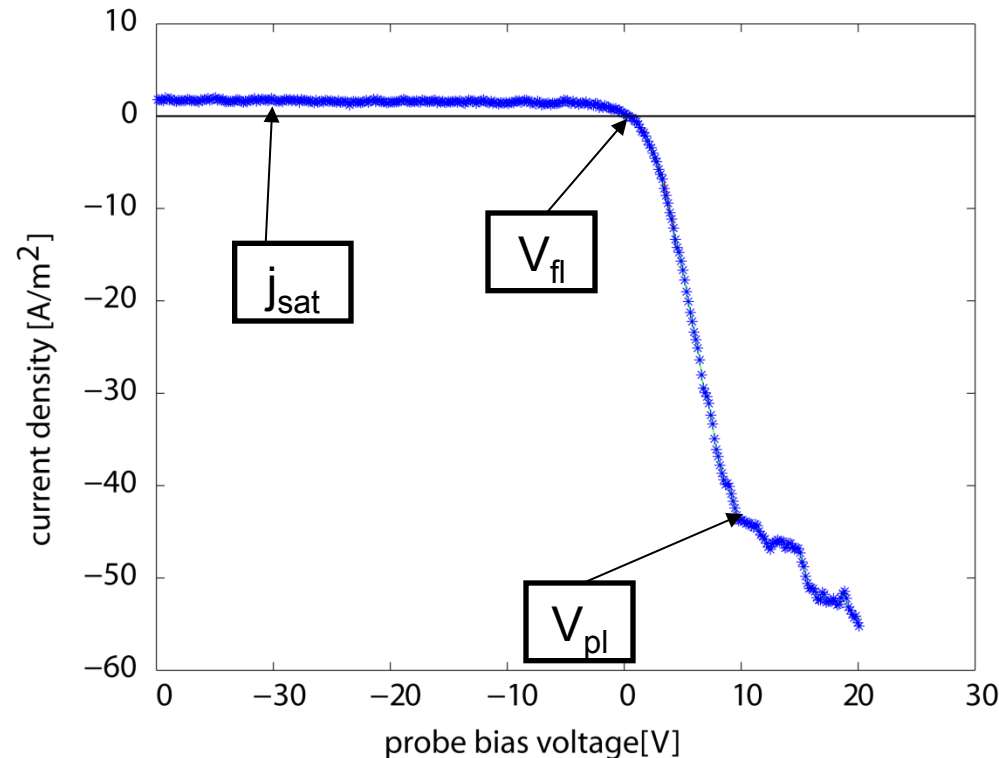
I_{sat} : ion saturation current. Often we work directly with $J_{sat} = I_{sat}/A_s$, with A_s the particle collection area – this is identified with A_{\perp} , the projected area when $B \neq 0$ (see later).

Once I_{sat} and T_e are known, the density **at the sheath edge** follows:

$$n_{se} = I_{sat} / Z_i e c_s A_s$$

However, the knee it is not well defined (see practicum III).

Usual way to extract information from LP characteristic is to use a 3 parameter non-linear least square fit to obtain I_{sat} , V_f and T_e .

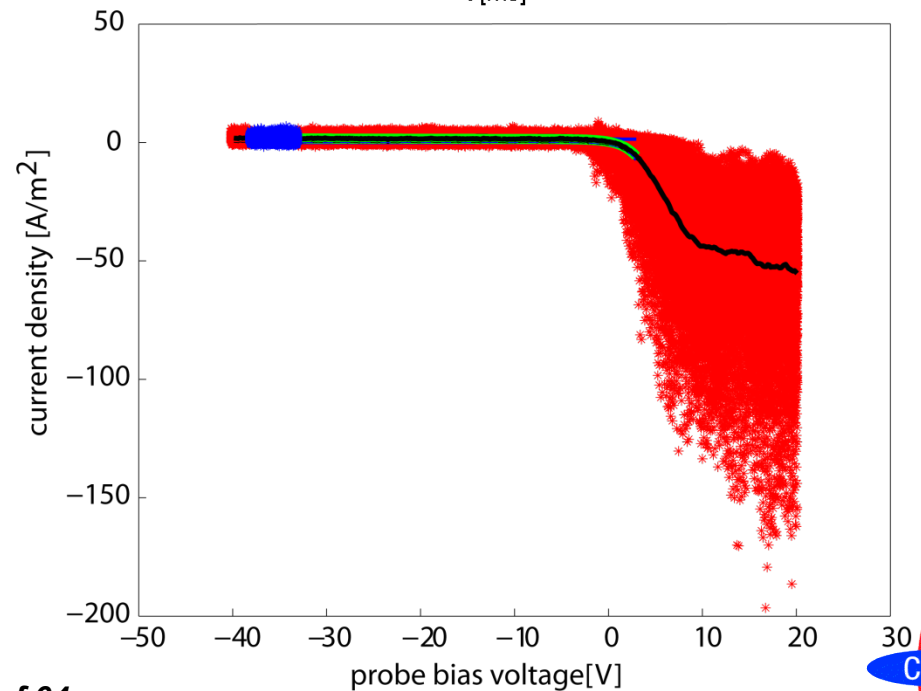
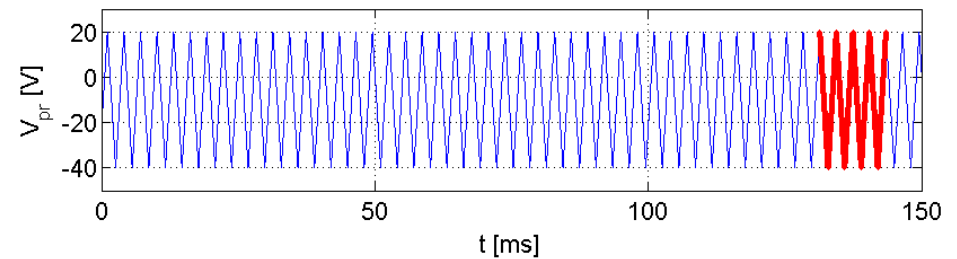
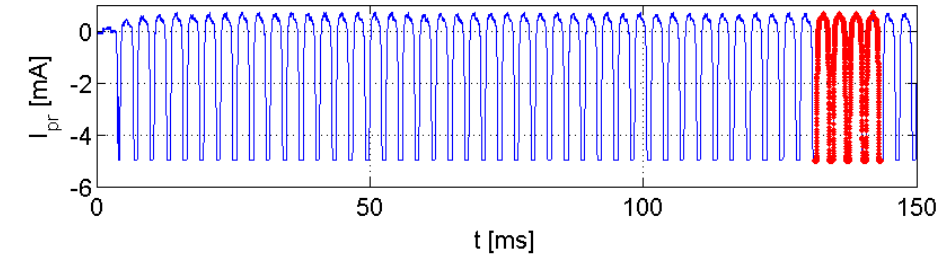
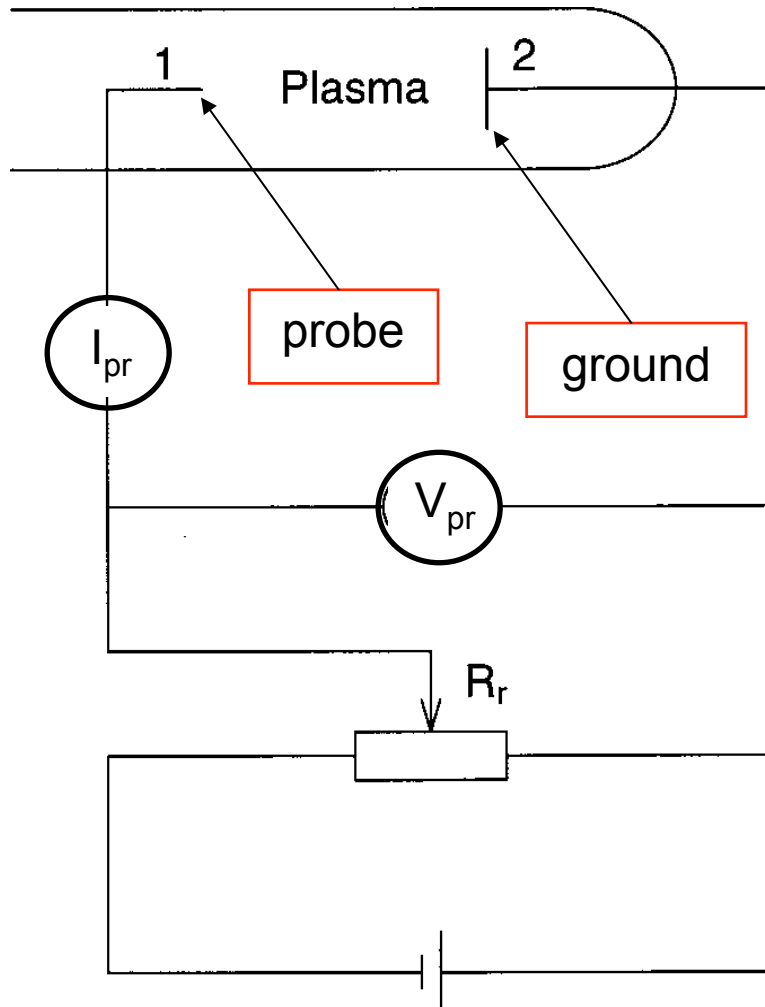


$$I = I_{sat} \left[1 - \exp \left(- \frac{V - V_{fl}}{T_e} \right) \right]$$

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Single LP in practice: sweeping the probe



Practical consideration - I: sheath expansion

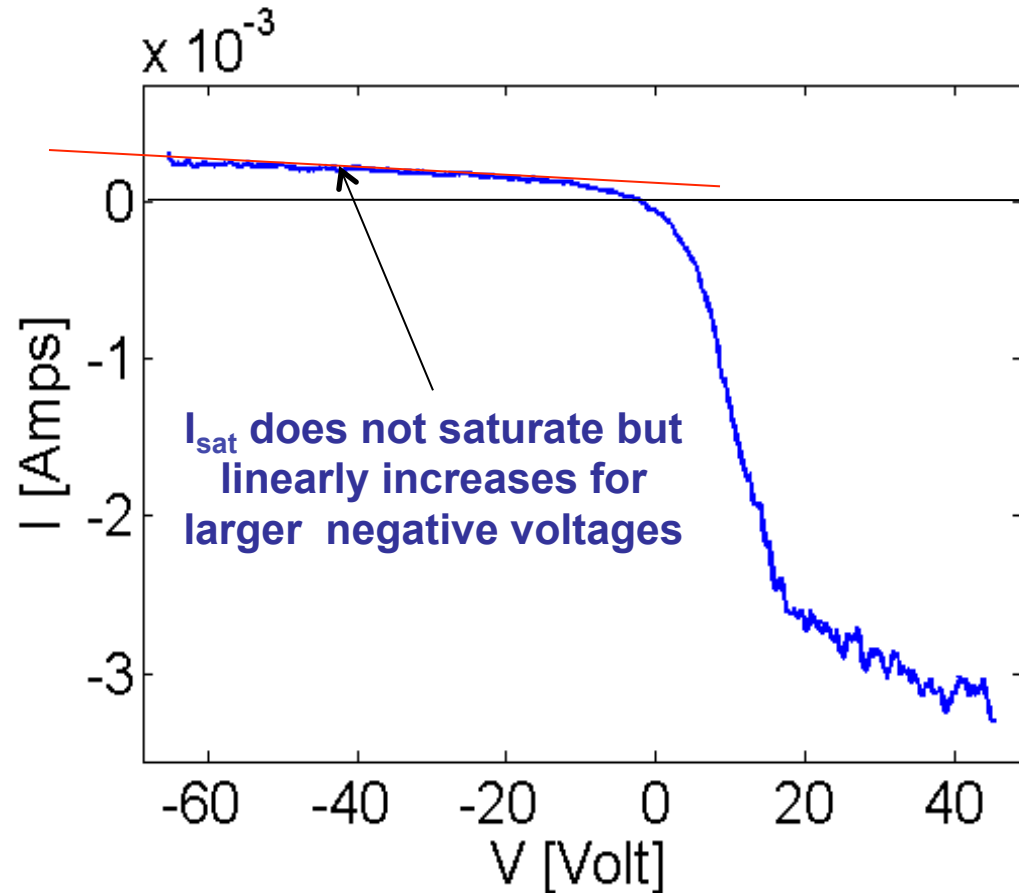
The sheath is usually 15-30 Debye length thick $\lambda_d = 7.43 \times 10^2 (T_e [\text{eV}] / n_e [\text{cm}^{-3}])^{1/2} \text{ cm}$

However the sheath expands at large negative voltage (Hutchinson) \rightarrow increase of the collection area with negative biasing voltage $\rightarrow I_{\text{sat}}$ does not saturate.

$$x_s / \lambda_d = 1.02 [(-eV_{\text{pr}} / T_e)^{1/2} - 2^{-1/2}]^{1/2} [(-eV_{\text{pr}} / T_e) + 2^{1/2}]$$

This effect **MUST** be taken into account when the size of the probe is comparable with the sheath thickness. In this case, the probe area must be corrected for the sheath thickness (spherical: $A_s = A_{\text{pr}}(1 + x_s/a)^2$, cylindrical $A_s = A_{\text{pr}}(1 + x_s/a)$).

This example is from a the FRIPLE probe in TORPEX.

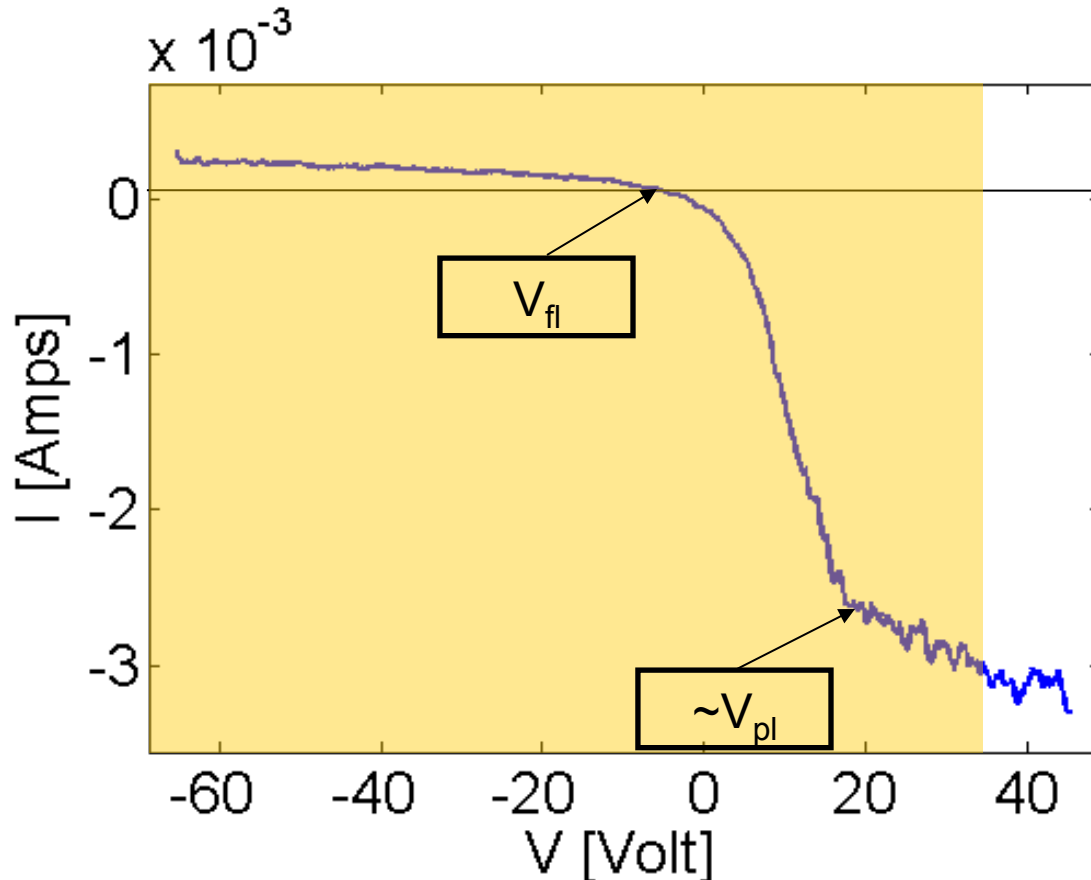


How to fit a Langmuir I-V curve

$$I = I_{\text{sat}} \left[1 - \exp \left(- \frac{V - V_{\text{fl}}}{T_e} \right) \right]$$

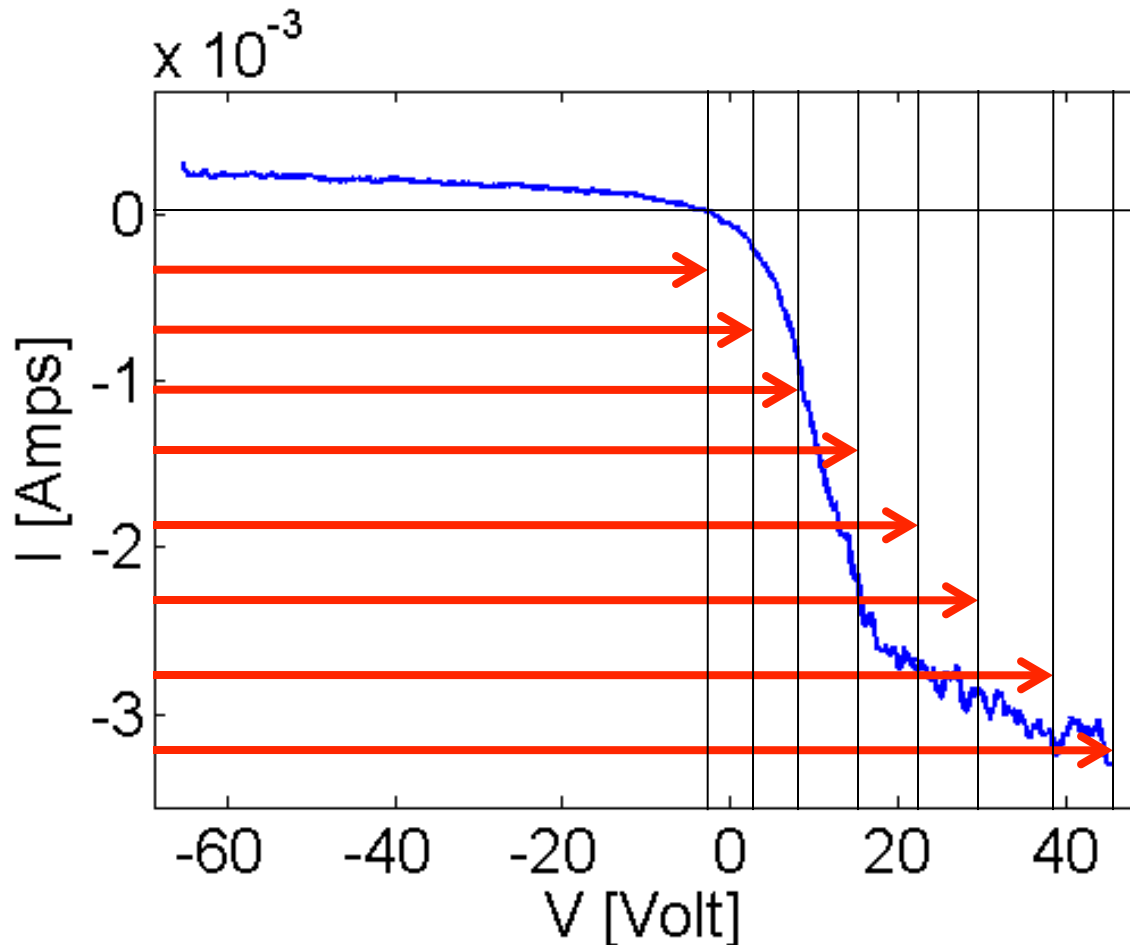
$$I = I_{\text{sat}} \left[1 - \alpha (V - V_{\text{fl}}) - \exp \left(- \frac{V - V_{\text{fl}}}{T_e} \right) \right]$$

Takes into account the sheath expansion



Problem in choosing the interpolation boundaries: interpolating over a range $V > V_{\text{pl}}$ (see figure) would result in an over-estimate of the electron temperature.

Minimum temperature method



determine V_{fl}



interpolate the I-V curve from:

$\min(V) \rightarrow V_{fl}$

$\min(V) \rightarrow V_{fl} + \Delta V$

...

$\min(V) \rightarrow \max(V)$



minimum T_e provides:

$T_{e,g}, V_{fl,g}$



repeat interpolation over:

$[vm_4_par * T_{e,g} + V_{fl,g}, vM_4_par * T_{e,g} + V_{fl,g}]$

Practical consideration - II: bandwidth limitation

Measuring fast fluctuations of plasma parameters requires reconstructing many LP characteristics per unit time. This requires sweeping the LP tip at high frequency. This has some limitations (see also Exercise I on Tuesday).

$$\left. \begin{aligned} I &= I_{\text{sat}} \left[1 - \exp \left(-\frac{V - V_{\text{fl}}}{T_e} \right) \right] \\ \frac{V - V_{\text{fl}}}{T_e} &\ll 1 \end{aligned} \right\} \begin{aligned} I &= I_{\text{sat}} \frac{\Delta V}{T_e} \\ R_{\text{sheath}} &= \frac{T_e}{I_{\text{sat}}} \end{aligned}$$

Near the floating potential the sheath acts as a resistor with a resistance R_{sheath} . Therefore the sweeping frequency is limited by capacitive coupling (cables, plasma) \rightarrow time resolution is limited; no fast fluctuations.

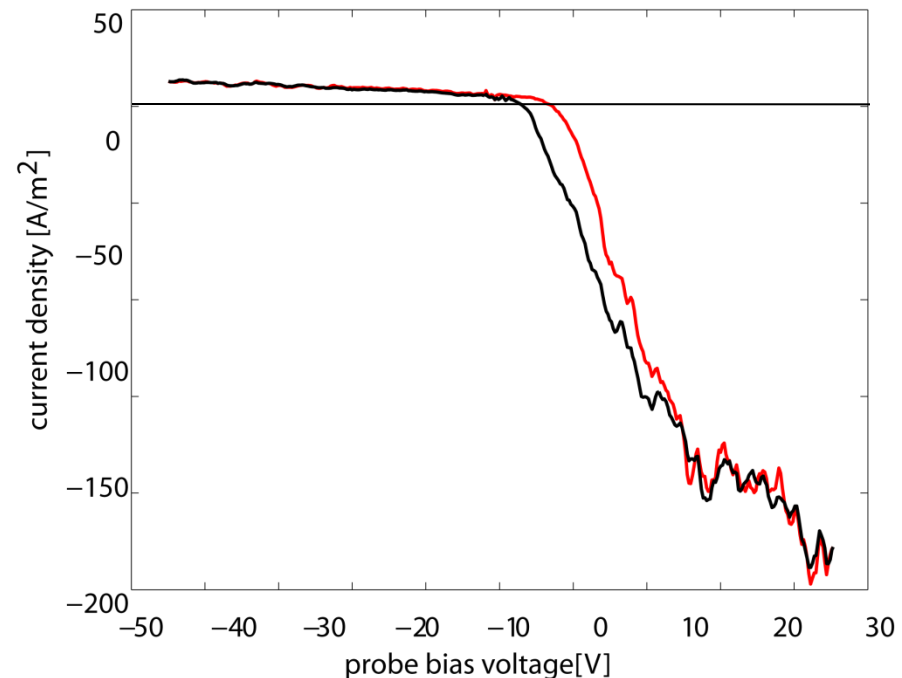
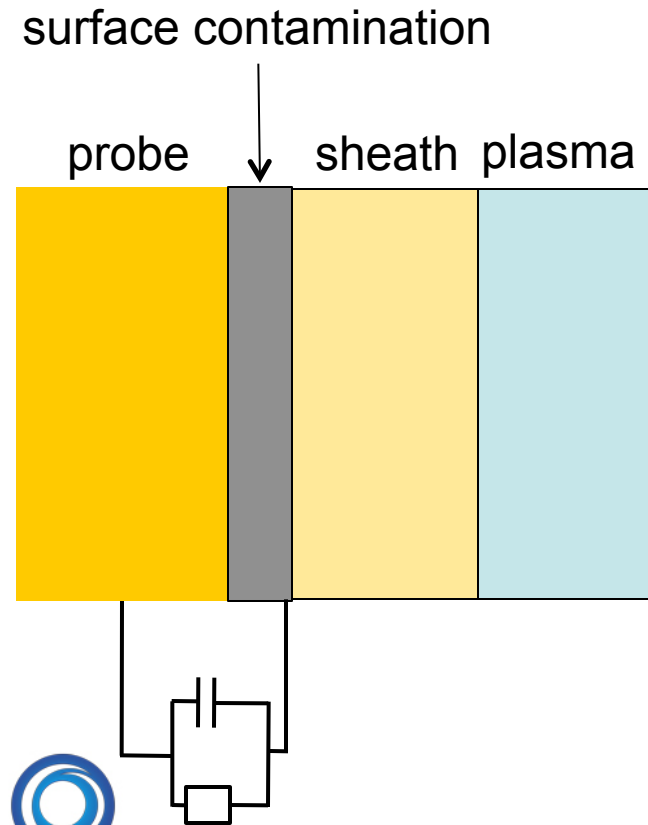
$$R_{\text{sheath}} [\text{Ohm}] \approx 1.27 \times 10^{15} \frac{\sqrt{T_e [\text{eV}] m_i / m_p}}{A_{\text{probe}} [\text{m}^2] n_e [\text{m}^{-3}]}$$

Warning for experimentalists: probe contamination

Surface layer contamination may come from deposition of sputtered materials or from absorption of gases and vapors in the plasma itself. A cleaned probe, when immersed in a neutral gas, may immediately absorb ambient neutral species.

If these species are non conductive, this layer acts as an impedance. When the voltage is applied to the probe, charge ions flow to it and charged the capacitance and at the same time change the surface layer by bombardment. This changes the

work function of the probe and results in a hysteresis of the I-V curve during up-down sweep.



Probe contamination: possible solutions

The hysteresis may result to erroneous measurements especially for the floating potential. Therefore it should be avoided.

Ramp sweep (up or down exclusively) should also be avoided because they do not reveal the surface contamination problem.

Avoid surface contamination:

- Periodic cleaning of the probe by either ion bombardment (ion saturation mode) or electron bombardment (electron saturation mode). The effectiveness of the method depends on the absorption rate (difficult to determine) and sometimes the clean up is necessary right before the measurements.
- Periodic heating of the probe (this involves a specific design of the probe)
- Fast pulsed-voltage cycle of the modulation (E. Szuszczewicz, J. Appl. Phys. 46, 12 1975)

On TORPEX, the ion bombardment method is the most effective. Typically the probe is run in ion saturation mode for a few minutes.

Limits of collisionless theories with $B \neq 0$

The main effect of the magnetic field is to cause both electrons and ions to move no longer in straight lines but to spiral around the field lines in circular orbits of radius $\rho = mv/eB$.

If $\rho_{i,e} > a$, where a is the typical dimension of the probe then the previous treatment applies (on TCV: $\rho_i \sim 1$ mm, $\rho_e \sim 0.01$ mm, on TORPEX: $\rho_i < 1$ mm, $\rho_e \sim 0.05$ mm).

If $\rho_i > a$, $\rho_e < a$, then the electrons are more affected than the ions and the electron flow is impeded \rightarrow reduction of the electron saturation current. If the probe is significantly negative so that most of the electrons are reflected (ion collection) then the previous theory still provides adequate results.

If $\rho_{i,e} < a$, significant modifications to the ion collection can occur. A complete theory is still missing. The practical approach is the following. The electrons are still considered to be governed by the Boltzmann factor when $V < V_{fi}$ and therefore the temperature can be deduced from the slope of the I-V curve (see next slides). Ions are still considered to flow out at the ion sound speed along the field lines, therefore the effective area is computed by using the projection of the surface in the direction of the magnetic field.

i-e current ratio and over-estimation of T_e when $B \neq 0$

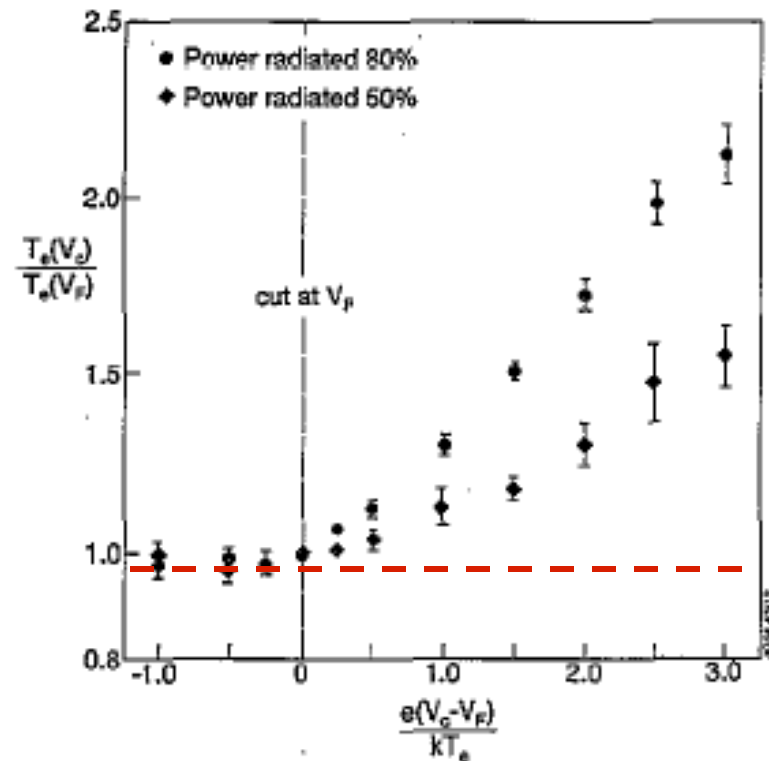
Normally expect $E_{\text{sat}}/I_{\text{sat}} \approx (m_i/m_e)^{1/2} \approx 60$ for D^+ plasma.

In a magnetized plasma with $\rho_e \ll a$, $E_{\text{sat}}/I_{\text{sat}} \sim 10$ is usually observed and, for low T_e , strongly detached divertor plasmas, $E_{\text{sat}}/I_{\text{sat}} \sim 1$ is found.

The lower than expected “normal” ratio (i.e. ~ 10) is due principally to the restricted electron motion in strong magnetic fields.

Characteristic “deviates” from the simple exponential for values of $V_{\text{pr}} > V_f$ – electron current above V_f increases more slowly than expected \rightarrow fitted T_e higher than it should be.

However, the ion saturation current does not change \rightarrow theory still valid.

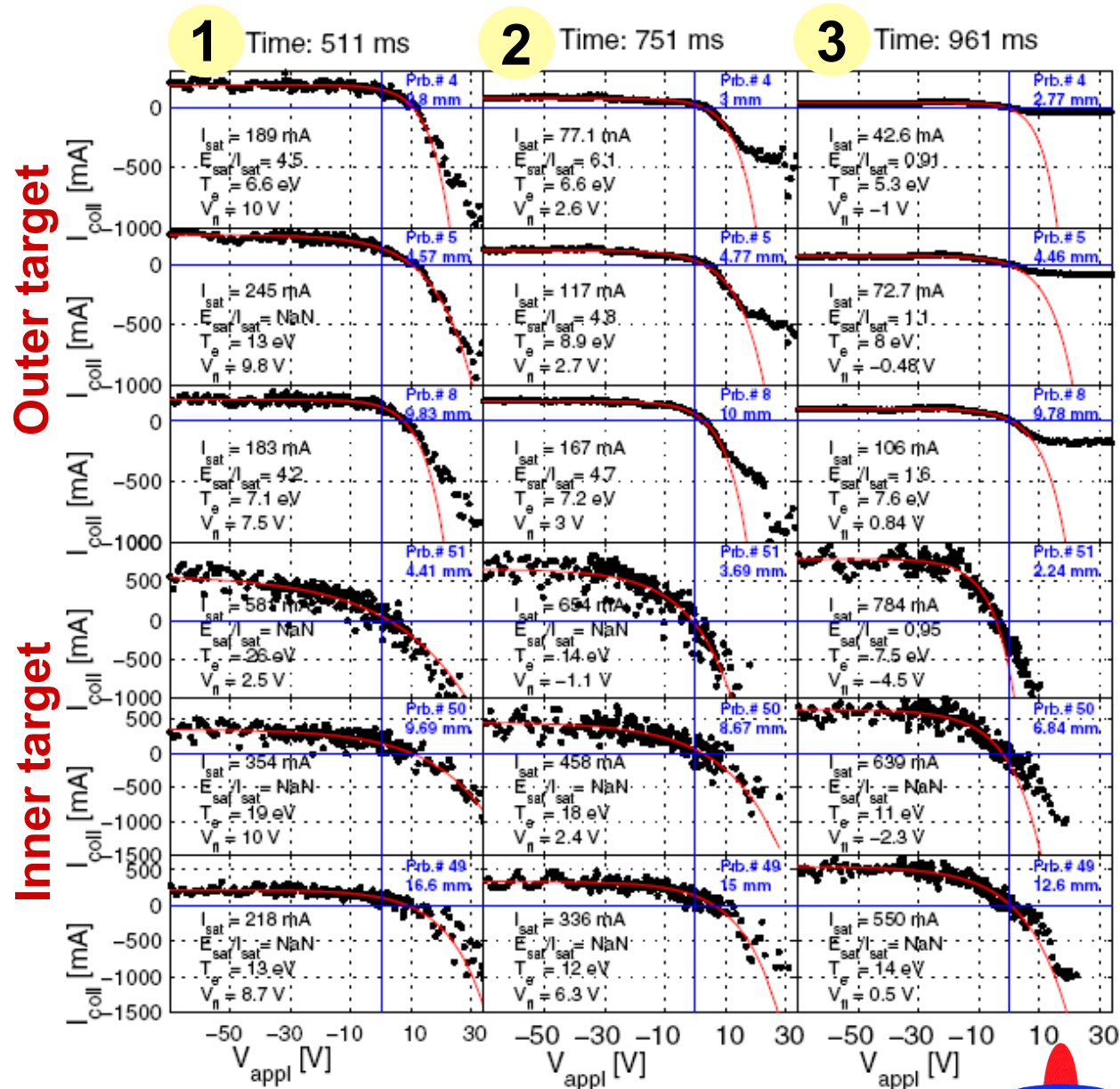
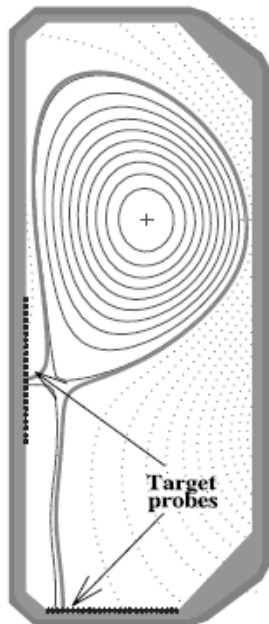


T. Tagle et al., PPCF 29 (1987) 297

Example LP characteristics from TCV

TCV divertor probes provide a good example of the fitting problems that can be experienced in magnetic fields

- 1: Low recycling
- 2: High recycling
- 3: Detached(outer) divertor



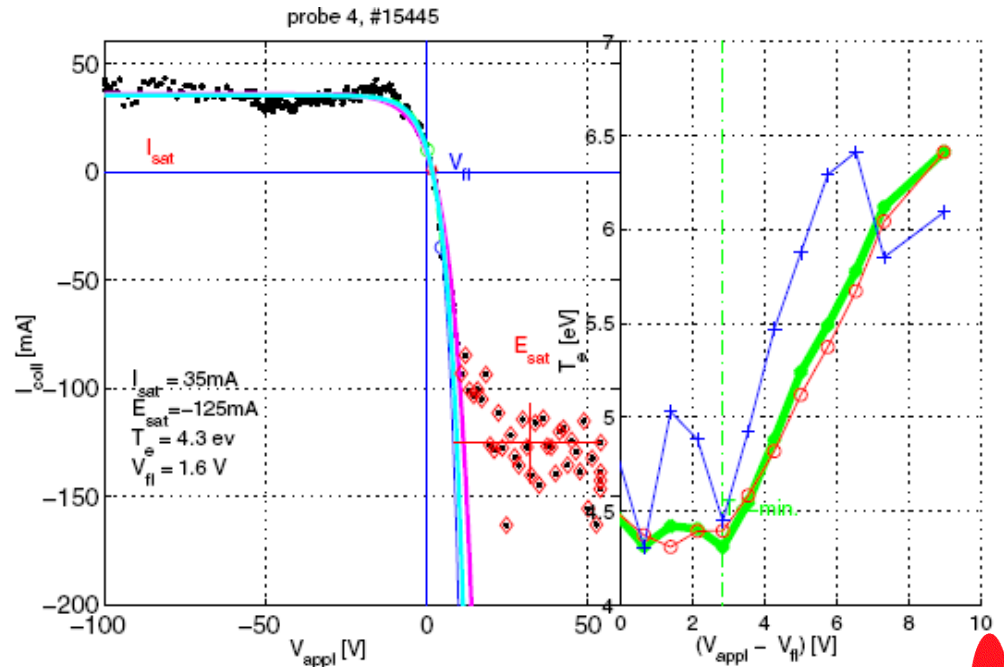
Particle collection when $B \neq 0$

Message is therefore clear: in a strongly magnetized plasma, do not use the region $V_{pr} > V_f$ when fitting the Langmuir probe I-V characteristic to extract T_e !!!

In practice, a few points on the electron collection side of V_f are usually required to better constrain the 3-parameter non-linear fit.

Minimum temperature method: make multiple fits to progressively fewer points on the electron collection side until a “minimum” T_e is found and then accept this value.

An example from TCV

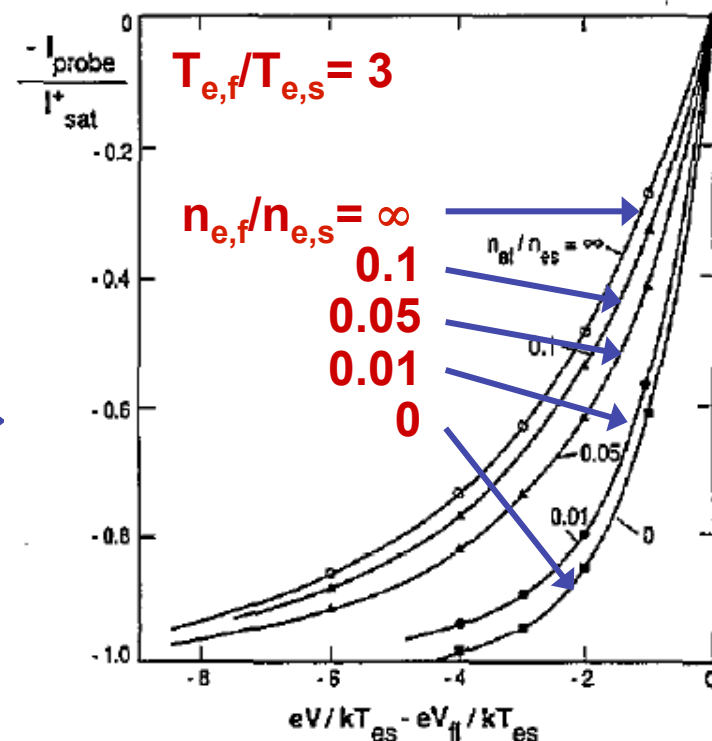


Only high energy electrons are sampled

If we restrict ourselves to fitting only the portion of the probe characteristic for which $V_{pr} < V_f$, then T_e is estimated only from that part of the electron distribution capable of overcoming the sheath potential fall ($\sim 3T_e$).

This also means that any fast electrons in the incoming electron flux can dominate the I-V characteristic.

For example, if the electron distribution sampled by the probe were a two component Maxwellian with $T_{e,f}/T_{e,s} = 3$, it does not require many fast electrons (say $> 5\%$ at $T_{e,f}$) to produce an I-V characteristic which would be experimentally indistinguishable from a curve to which a fit would yield $T_e = T_{e,f}$



Theoretical characteristics: I_{pr}/I_{sat} vs. $(V_{pr}-V_f)/T_{e,s}$ for $V_{pr} < V_f$ for an incoming electron flux with varying fast electron density ratio: $n_{e,f}/n_{e,s}$

P. C. Stangeby., PPCF 37 (1995) 1031

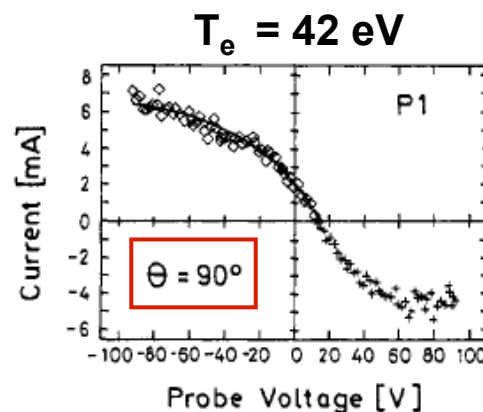
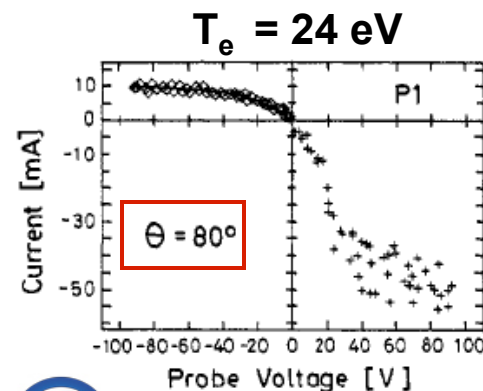
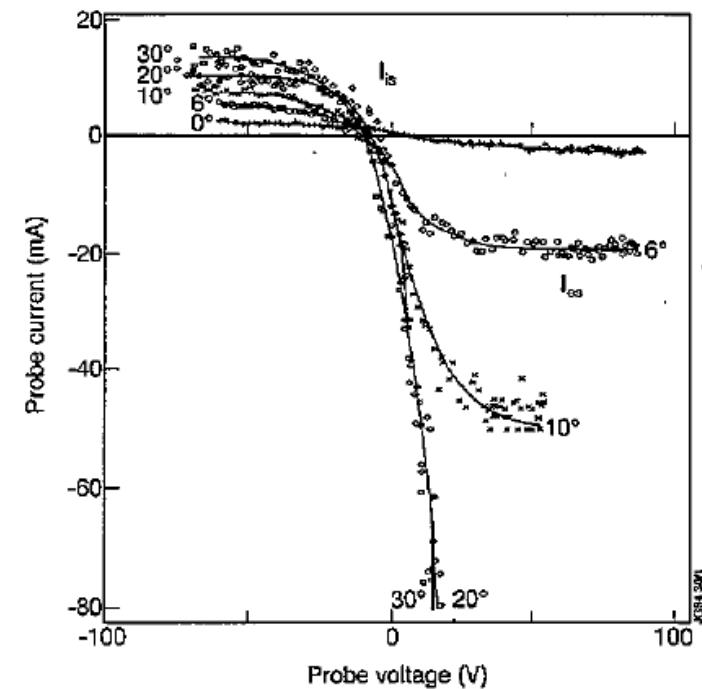
Grazing magnetic field line incidence

For probes embedded in divertor target tiles, and particularly those which are **flush mounted**, the problem of grazing magnetic field line incidence can be an issue: typical angles to the surface in divertors are in the range $\alpha = 0.5 \rightarrow 7^\circ$ (G. F. Matthews et al., PPCF 32 (1990) 1301)

There is currently no reliable theory for flush mounted probes at grazing incidence.

Experiments using a “tilting probe” on the DITE tokamak showed that:

- 1) Ratio $I_{\text{sat}}/E_{\text{sat}}$ strong function of α .
Ratio \rightarrow unity for grazing incidence
- 2) I_{sat} no longer saturates at small $\alpha \rightarrow$ increases linearly with V_{pr}
- 3) Fitted T_e too high for flush mounted

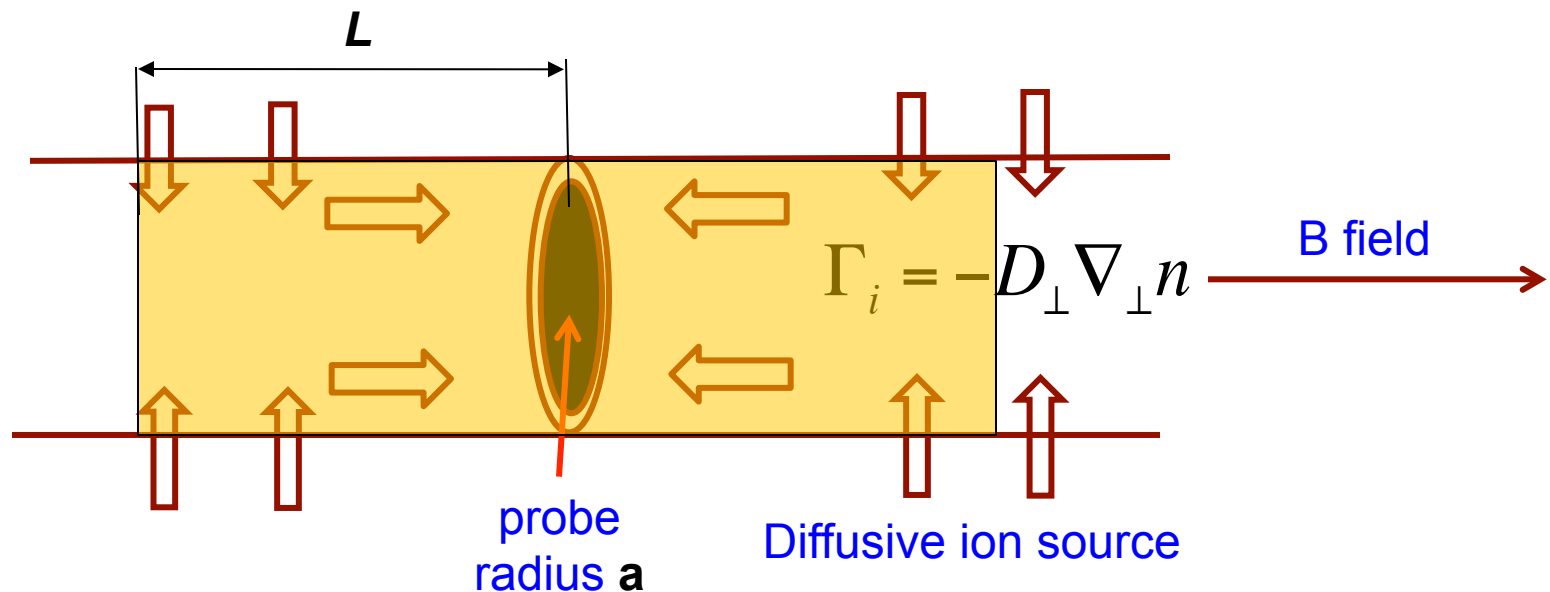


Collisions when $B \neq 0$

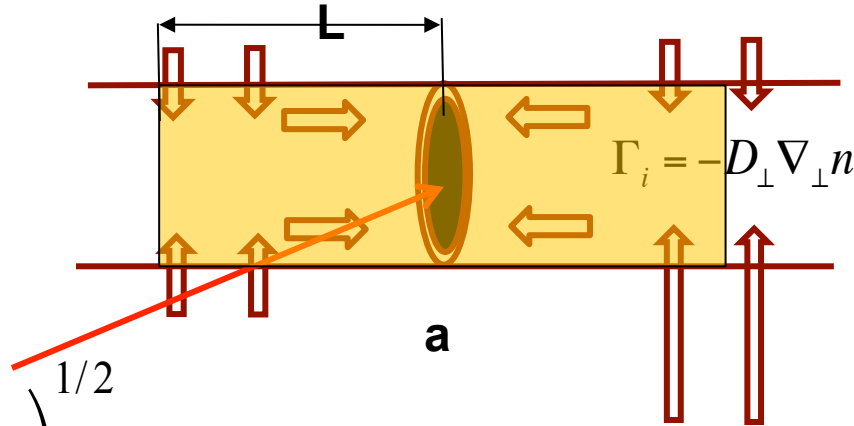
Is the collisionless model valid? For this to happen, the ions mean free path along the magnetic field, l , must be larger than the length of the probe collection (pre-sheath) region, L .

We note that only collisions between ions and other species (i.e. electrons, neutrals, ...) are important since ion-ion collisions, though they change the ion distribution function, they do not change the total ion momentum.

The length of the collection region must be great enough to allow sufficient ion sources to replenish the flux tube which is emptied by the flow to the probe.



Quasi-collisional theory in magnetic field



$$j_i \approx \frac{1}{2} n_0 \left(\frac{T_e}{m_i} \right)^{1/2} \underbrace{\pi a^2}_{L = \frac{a^2}{D_{\perp}} \left(\frac{T_e}{m_i} \right)^{1/2}} \quad j_{\perp} = 2\pi L a D_{\perp} n_0 / 4a$$

$$L = \frac{a^2}{D_{\perp}} \left(\frac{T_e}{m_i} \right)^{1/2}$$

Bohm diffusion

$$D_{\perp} = T_e [\text{eV}] / 16B$$

For example in TORPEX, charge exchange collisions with neutral gives $L \sim 0.4$ m. While $L \sim 5$ cm, therefore we can use a collisionless approach.

Content of unit

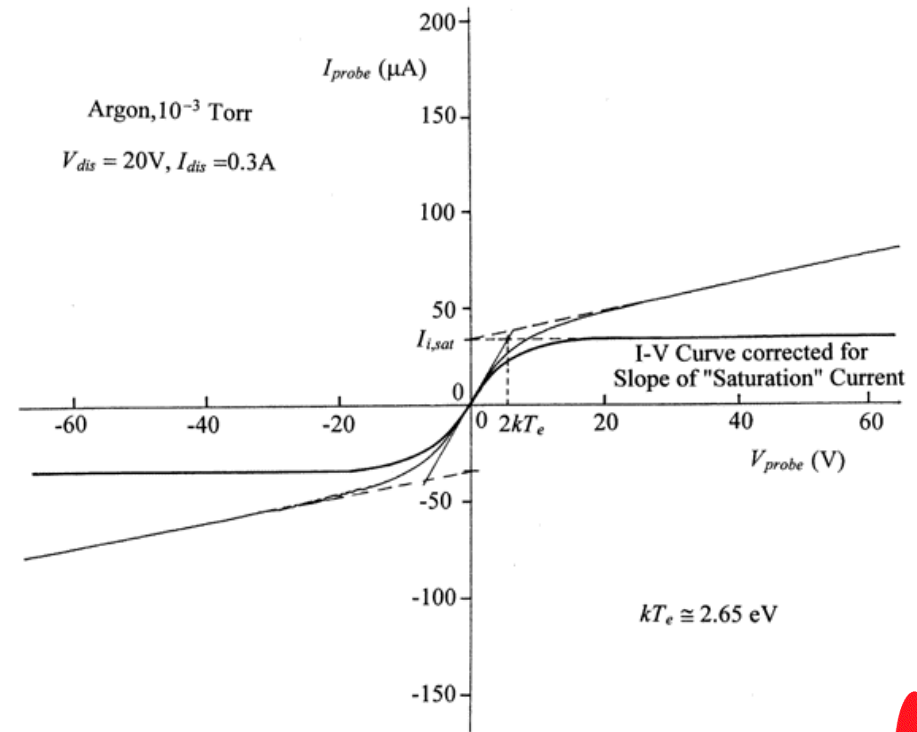
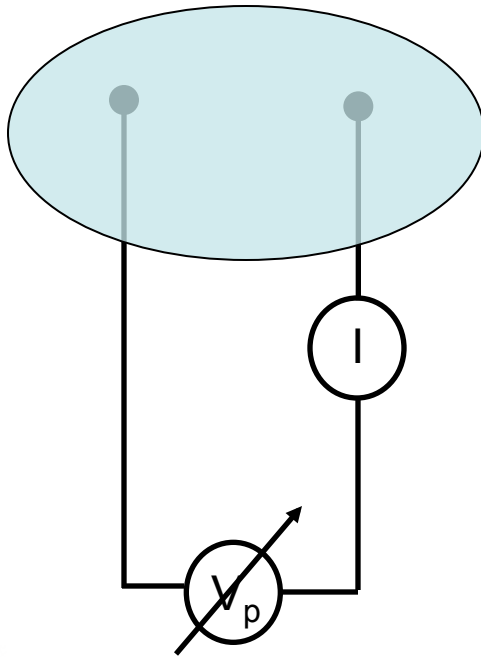
- Langmuir probe (LP) examples
- Theory of single LP
 - Analysis of the sheath
 - Fluid plasma description
 - The I-V characteristic
 - How to extract plasma parameters
- Langmuir probes in practice
 - How to implement a LP
 - Typical experimental problems and how to face them
 - Sheath expansion, bandwidth limitation, probe contamination
 - Langmuir probes in magnetic fields
- **More complex electrostatic probes**
 - Double LP, Triple LP, harmonic method for T_e
 - Katsumata, ballpen probes
- **Electrostatic analyzers**
 - Druyvesteyn method for $f(v)$ measurements, grid energy analyzer

Double probe principle

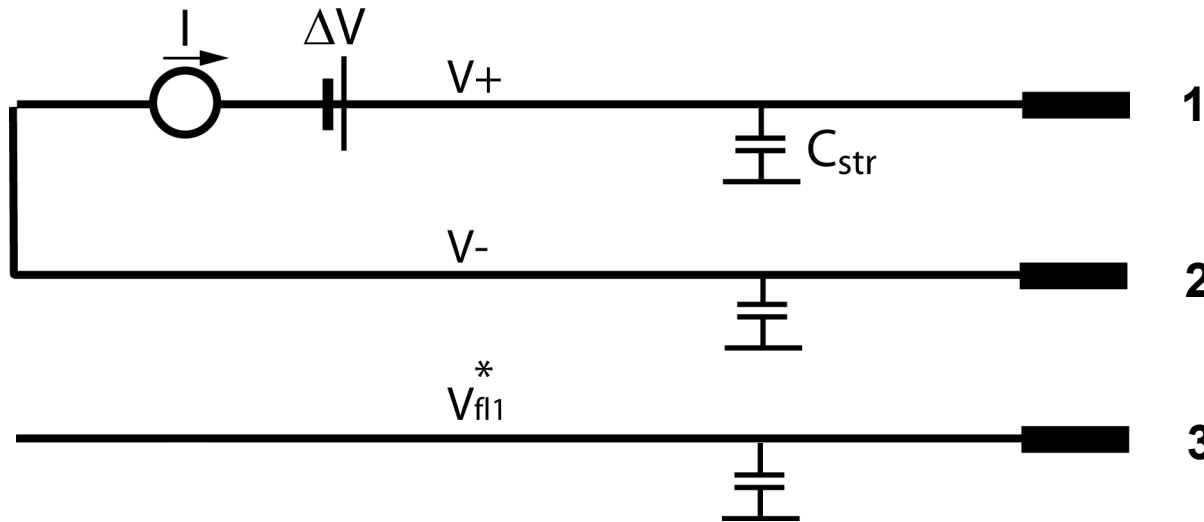
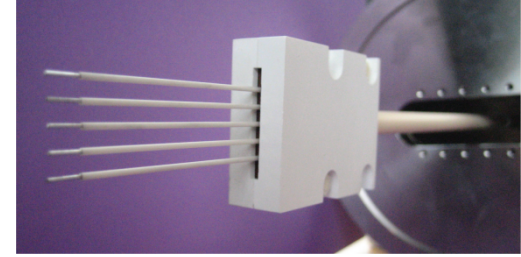
Single LP probe requires well defined ground → difficult in spacecrafts (ionosphere). In electron saturation mode, the electron current can be large → damage to probe.

Double probes do not require well defined ground. For negative voltages, I_{sat} is the maximum current → no damages to probe.

The I-V characteristics is given by:
$$I = I_{\text{sat}} \tanh\left(\frac{V}{T_e}\right)$$



Triple probe principle



Two tips (1-2) are floating and a bias $\Delta V \gg T_e/e$ is applied between them.

The third tip is also floating and measuring V_{fl} .

The triple probe provide direct time-resolved measurements of the electron temperature: no need to sweep the probe. This also provides direct time-resolved measurements of the plasma potential, which is needed for turbulent flux measurements (see Exercise-I on Tuesday).

$$T_e = \frac{V^+ - V_{fl}}{\log(2)}$$

$$n_e = \frac{2I}{ec_s A}$$

$$V_{pl} = V_{fl} + \Lambda T_e$$

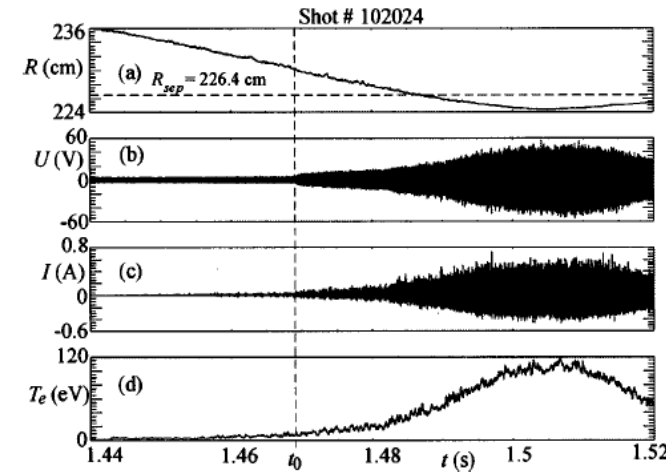
Harmonic method for T_e measurements

Harmonic method for fast T_e measurement has been used successfully on TEXTOR and DIII-D.

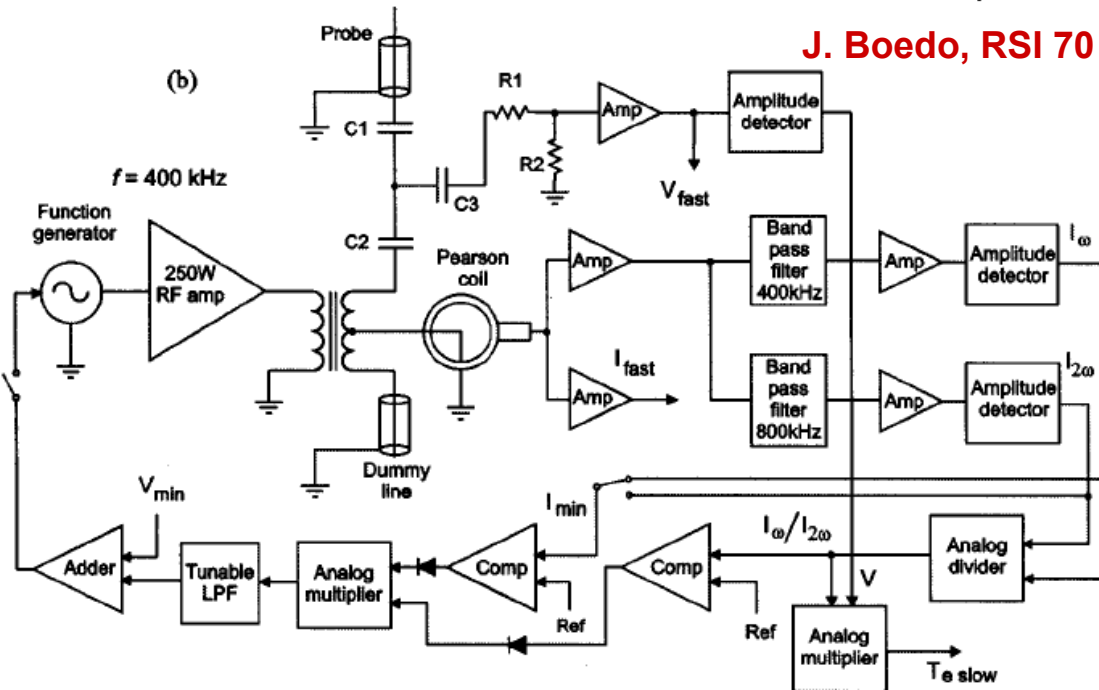
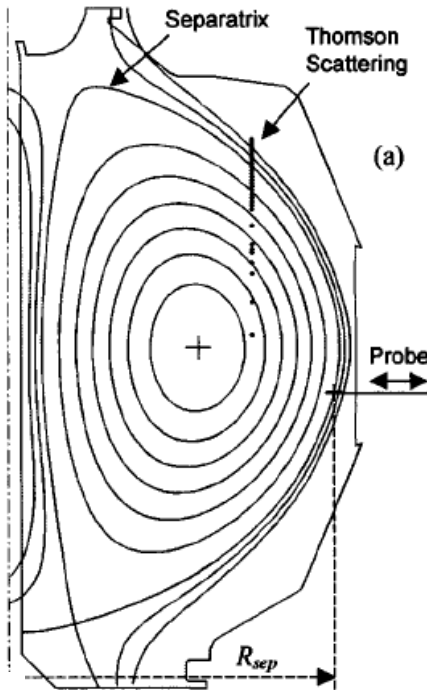
T_e determined from ratio of 1st and 2nd harmonics in the probe sheath response to a fast sweep (400 kHz):

$$T_e = \frac{eU_0}{4} \frac{I_\omega}{I_{2\omega}}$$

where U_0 is the voltage sweep amplitude. Relation is valid up to $eU_0/kT_e \sim 1$ and works best if $0.5 < eU_0/kT_e < 1$.



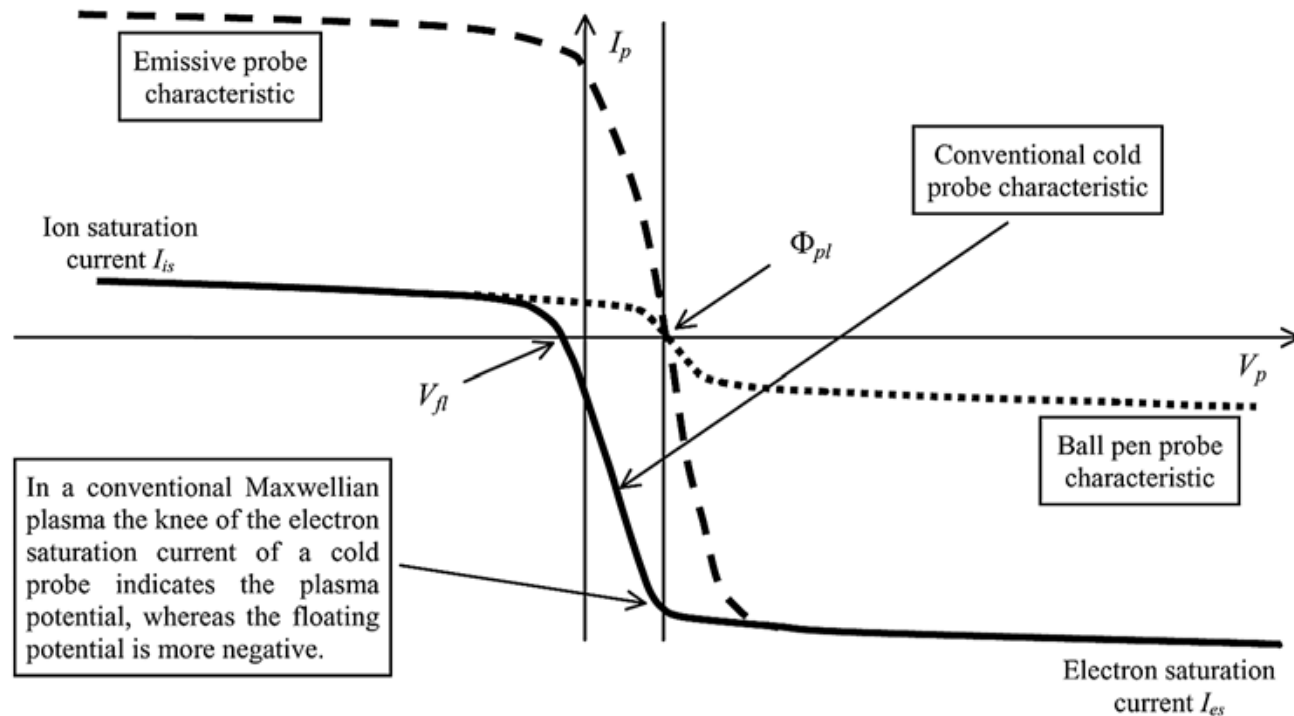
J. Boedo, RSI 70 (1999) 2997



Other types of LP for V_{pl} measurements

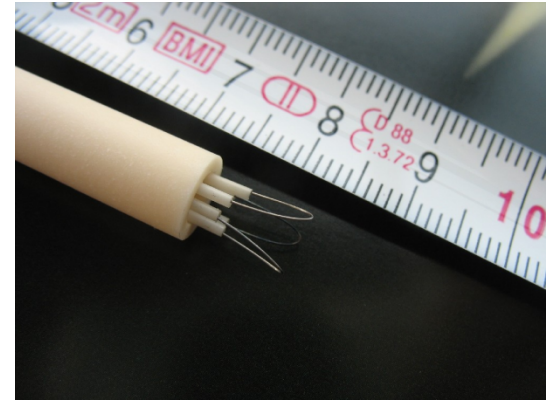
The main idea behind these types of probes (ballpen/Katsumata, emissive probes) is that by reducing the electron/ion saturation current ratio ($I_{e,sat} \rightarrow I_{i,sat}$) the floating potential moves towards the plasma potential.

$$V_{pl} = V_{fl} + T_e \log\left(\frac{I_{e,sat}}{I_{i,sat}}\right)$$

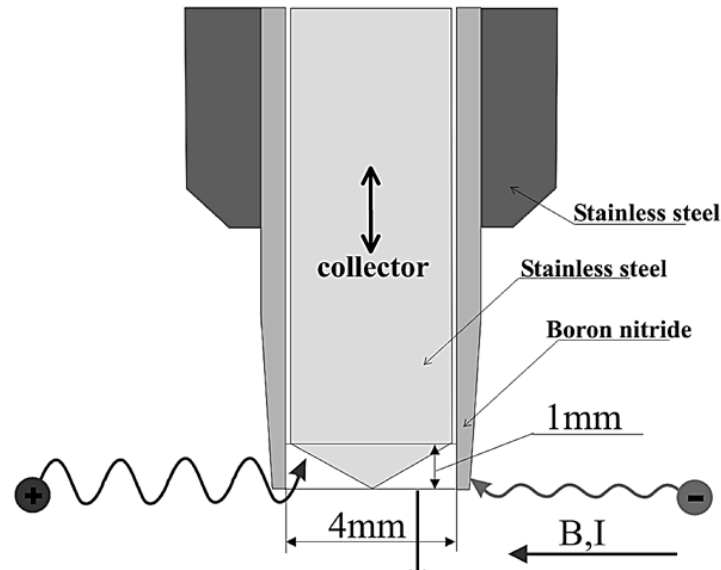


Emissive and Katsumata/ballpen probes

Emissive probes: the ion saturation current is increased by heating the probes that now emits electrons. They require quite complicated circuits for heating/measurement and precise tip machining to increase the tip resistance and thus Ohmic heating.



Ballpen: in the presence of a magnetic field the electron current is reduced by acting on the electron collection area. Used on many fusion device (Asdes, RFX, Castor). Requires precise study of the characteristics as a function of the collector distance.



Electron energy distribution function (EEDF)

$$I_e = eA_p \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{v_{\min}}^{\infty} f(v) v_z dv_z \quad v_{\min} = \sqrt{\frac{2e(\phi_{pr} - V_p)}{m_e}}$$

$$I_e = eA_p \int_{v_{\min}}^{\infty} dv \int_0^{\vartheta_{\min}} d\vartheta \int_0^{2\pi} d\varphi v \cos(\vartheta) v^2 \sin(\vartheta) f(v) \quad \vartheta_{\min} = \cos^{-1} \frac{v_{\min}}{v}$$

$$I_e = \pi e A_p \int_{v_{\min}}^{\infty} dv \left[1 - \frac{v_{\min}^2}{v^2} \right] v^3 f(v)$$

$$I_e = \frac{2\pi e^3}{m_e^2} A_p \int_V^{\infty} \varepsilon \left[\left(1 - \frac{V}{\varepsilon} \right) f(\varepsilon) \right] d\varepsilon$$

M. Druyvesteyn and F. Penning, *Reviews of Modern Physics*, vol. 12, no. 2, 87, (1940).

Harmonic method for EEDF measurements

$$\frac{d^2 I_e}{dV^2} = \frac{2\pi e^3}{m_e^2} A_p f[v(\varepsilon)]$$

$$\varepsilon = \frac{1}{2} m v^2$$

$$F(\varepsilon) d\varepsilon = 4\pi v^2 f(v) dv$$

$$f(v) = \frac{m_e^2}{2\pi e^3 A_p} \frac{d^2 I_e}{dV^2} \quad \text{Druyvesteyn formula} \quad F(\varepsilon) = \frac{4}{e^2 A_p} \sqrt{\frac{mV}{2e}} \frac{d^2 I_e}{dV^2}$$

Practically, a modulation $\delta(t)$ is superposed on the biasing voltage and the second harmonic component is measured which is proportional to the EEDF.

$$I(V) = I[V_0 + \delta(t)] = I(V_0) + \delta(t) \frac{dI(V_0)}{dV} + \frac{1}{2} [\delta(t)]^2 \frac{d^2 I(V_0)}{dV^2} + \dots$$

M. Druyvesteyn and F. Penning, *Reviews of Modern Physics*, vol. 12, no. 2, 87, (1940).

Particle Energy analyzers - I

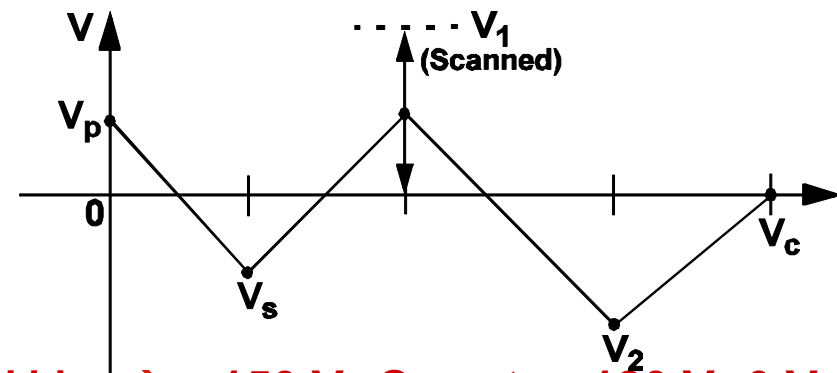
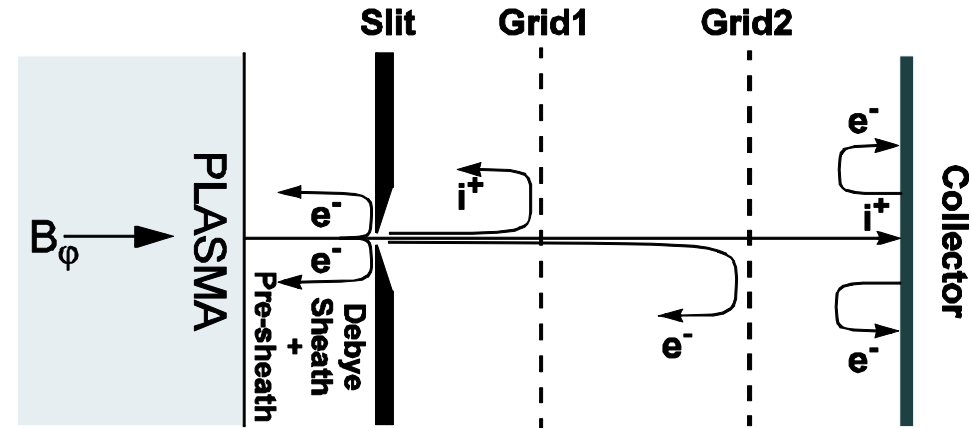
Langmuir probes are simple and easy to implement, but are limited in what they can measure. Interpretation, as we have seen, can also be problematic.

A quantity of particular interest, though rarely measured, is the SOL edge ion temperature, $T_i \rightarrow$ ion energy determines the rate of physical sputtering and hence a part of surface impurity release \rightarrow plasma contamination

CXRS can provide the impurity ion T_i in the pedestal and core, but signal intensities low in the SOL.

Most commonly used technique to date is the Retarding Field Analyzer (RFA). Spectroscopic line broadening can also be used but is a line integrated measurement.

RFA principle – ion analysis



Typical bias \rightarrow -150 V Swept -180 V 0 V potentials

Slit in ion saturation (to draw in ions and reject primary electrons)

Particle Energy analyzers -II

RFA's measure the integral of the ion parallel velocity distribution up to a velocity, $u = (2q_i V_g / m_i)^{1/2}$ determined by the applied grid voltage V_g .

$$I_c = A q_i \int_u^{\infty} v_{||} f(v_{||}) dv_{||}$$

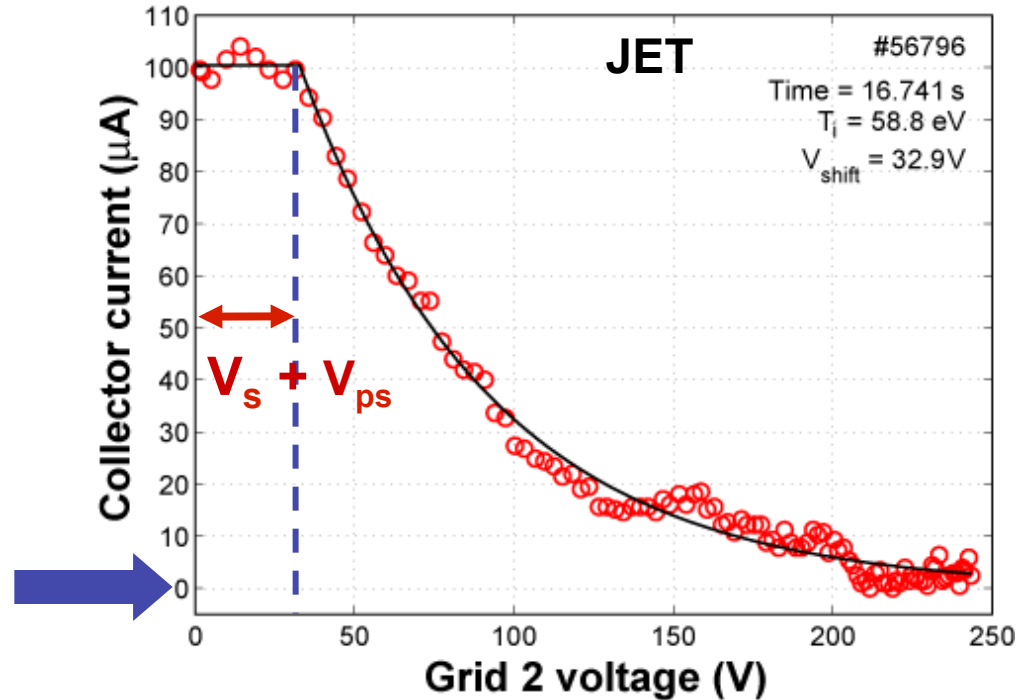
$f(v_{||})$ can in principle be obtained by numerical differentiation of the I-V characteristic. In reality, tokamak data are too noisy for this to be practical (difficult environment, small signals).

So, assume (experimentally justified) that $f(v_{||})$ is Maxwellian, shifted in velocity space by amount corresponding to the sheath and pre-sheath potential fall:

$$I_c(V_g) = A q_i \int_u^{\infty} v_{||} \exp(-m_i v_{||}^2 / 2eT_i) dv_{||} = I_i \exp(-Z_i V_g / T_i)$$

with $I_i = A Z_i T_i e^2 / m_i$, A is a normalization constant.

Typical RFA ion characteristic



Note that if the RFA electronics are referenced to the torus potential (which is usual), the local V_f must first be subtracted from the experimental value of V_s if one wishes to compare with the theoretical prediction for V

Particle Energy analyzers -III

RFA's are difficult to use in the tokamak edge:

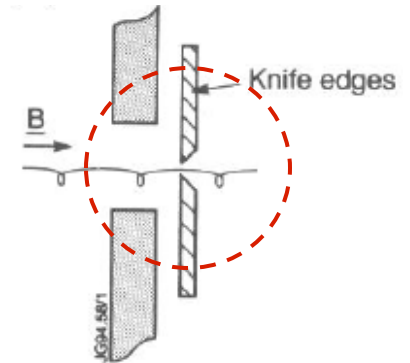
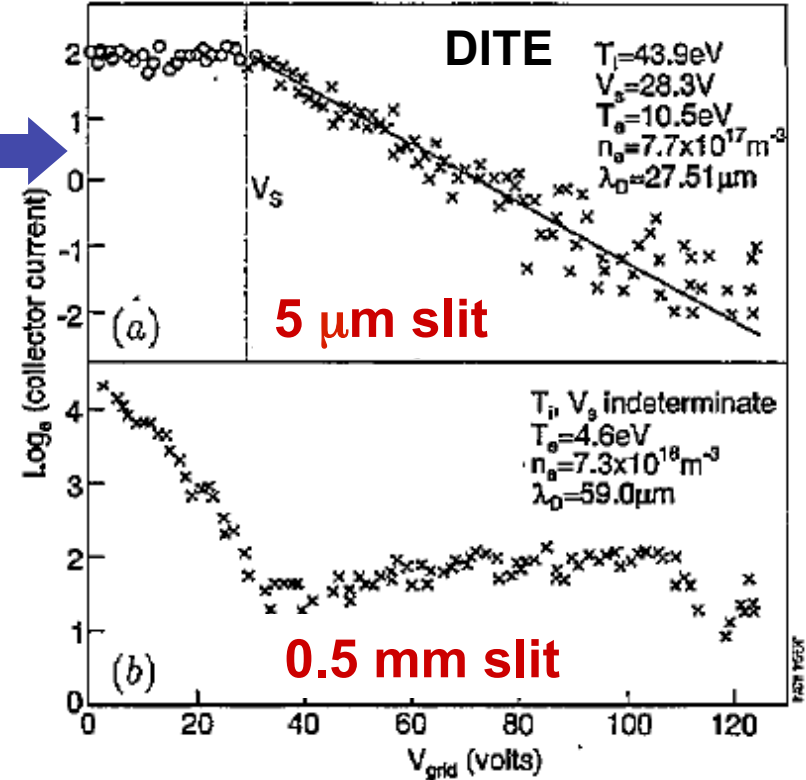
Require very thin entrance slits to ensure that slit plate sheath "bridges" the gap and so individual ion orbits are selected (recall sheath thickness of order $\lambda_D \sim 30 \mu\text{m}$ in the SOL) – thin slits which will tolerate high heat flux are difficult to manufacture

Require delicate, high transmission retarding grids

Thin entrance slit means small ion currents transmitted – μA of current at the collectors – this must be preamplified locally and excellent shielding required for in-probe cabling

In most tokamaks, fast reciprocation necessary to keep heat fluxes down \rightarrow fast sweeping for good time resolution \rightarrow a problem with capacitive coupling between planar grids

R. A. Pitts, Phd Thesis, Univ. London, 1991

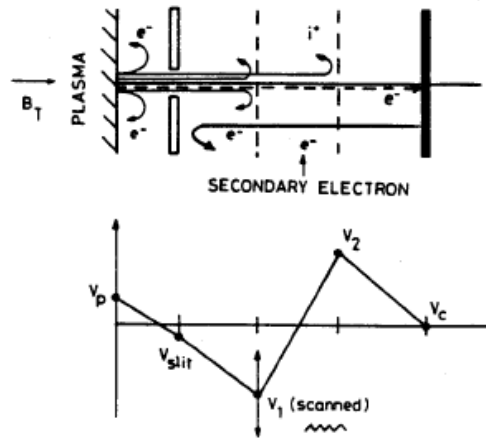


Particle analyzers – Electron retarding mode

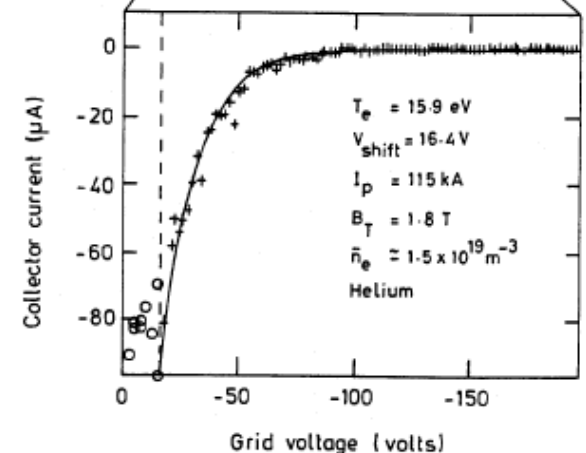
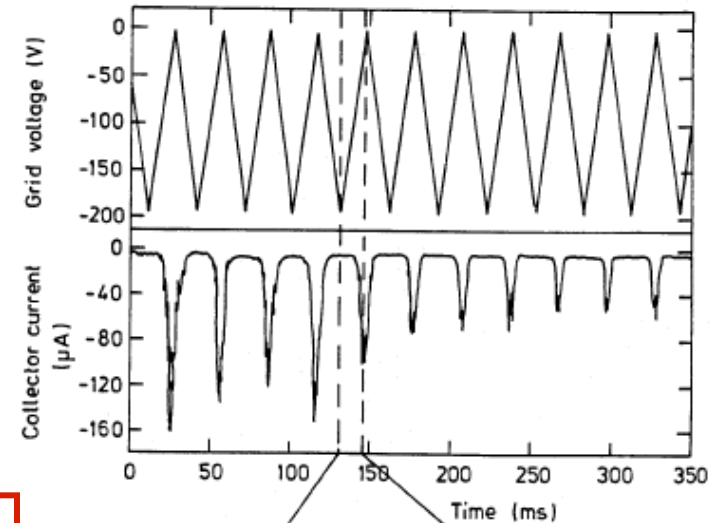
RFA's can also easily be operated in "electron retarding mode" providing now the integral electron parallel velocity distribution.

This is clearly less interesting, since T_e can also be accessed with a simple Langmuir probe.

BUT: the RFA does yield the full electron velocity distribution, with ions removed



RFA electron mode operation in DITE



R. A. Pitts, Phd Thesis, Univ. London, 1991

Particle analyzers – RFA (4)

JET RFA: an example of how to build one (1)

Designed (CRPP/JET), built CRPP

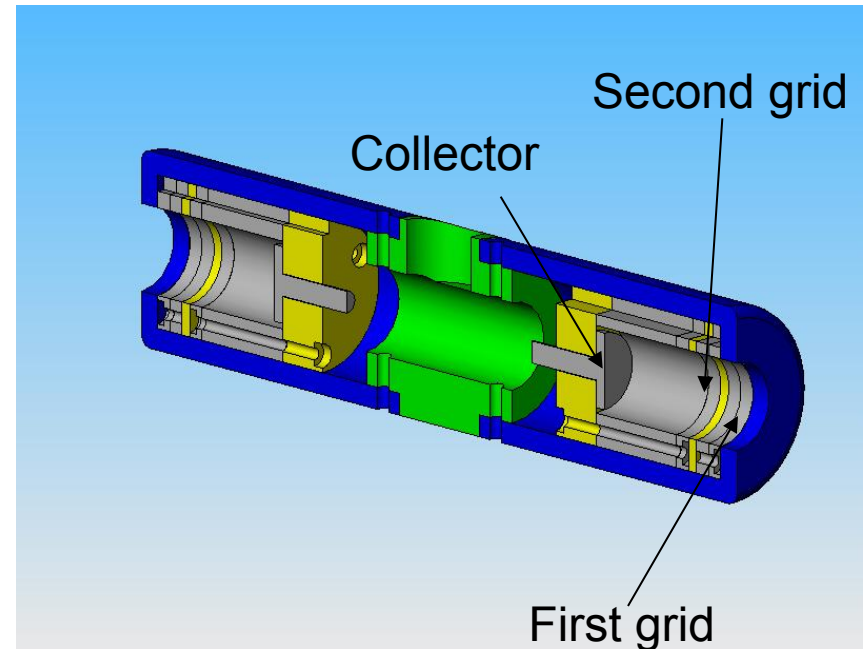
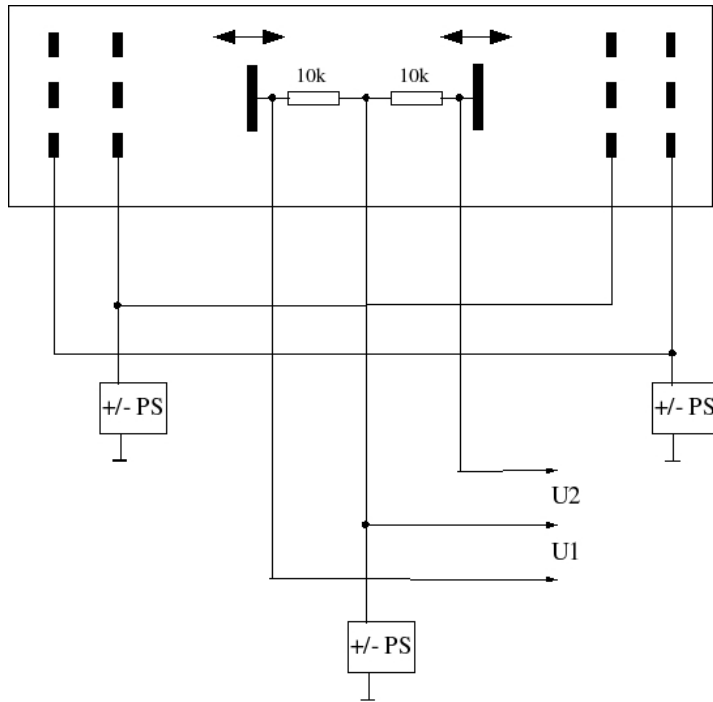


Particle analysers – RFA (5)

JET RFA: an example of how to build one (2)



Double gridded energy analyzer in TORPEX



Purpose: fast ions measurements ($E_{\text{ion}} \gg T_{\text{plasma}}$)
Two identical gridded energy analyzers for background subtraction

Literature

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F. F. Chen, Plasma Diagnostic Techniques, edited by R.H. Huddlestone and S. L. Leonard (Academic, New York, 1965).

J. D. Swift and M. J. R. Schwar, Electrical Probes for Plasma Diagnostics (Lliffe, London, 1970)