SIESTA and its free boundary development

H. Peraza-Rodríguez[1], R. Sánchez[1],J. Geiger[2], V. Tribaldos[1], J.M. Reynolds[1] and S.P. Hirshman[3]

[1] Universidad Carlos III de Madrid, Madrid, Spain
[2] Max-Plack Institut für Plasmaphysik IPP, Greifswald, Germany
[3] Oak Ridge National Laboratory, Tennessee, U.S.A.

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Outline

Table of Contents

SIESTA for newbies

What is SIESTA?

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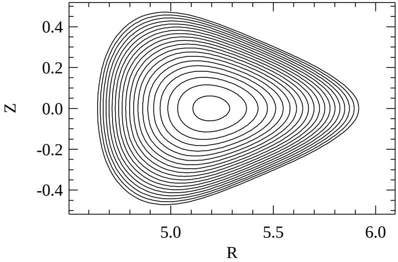
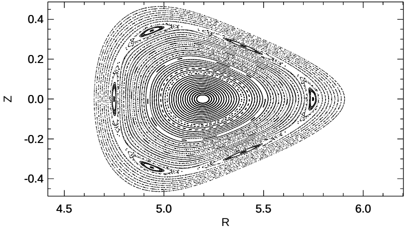
Scalable **I**terative **E**quilibrium **S**olver for **T**oroidal **A**pplications [1]

Ideal MHD code that introduces resistivity for the possible development of structures and allows a 3-D magnetic field.
Basically: improvement on VMEC code

VMEC



SIESTA



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[1] S.P. Hirshman *et al.*, Phis. Plasmas **18** 062504 (2011)

SIESTA's 'methodology'

Based on ideal MHD [+ resistivity and some tricks]:

- pressure: $\frac{\partial p}{\partial t} = (\gamma - 1)\mathbf{v} \cdot \nabla p - \gamma \nabla (p\mathbf{v})$ $\quad \rightarrow \quad$ $\xi = \mathbf{v}\Delta t$
- Faraday's law with resistivity:

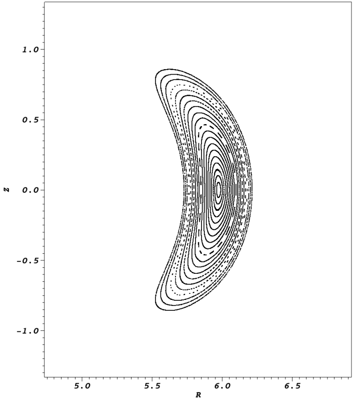
$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left[\mathbf{v} \times \left(\mathbf{E} + \begin{cases} \eta \mathbf{J} & \text{isotropic} \\ \eta \mathbf{J} \cdot \mathbf{B} \frac{\mathbf{B}}{B^2} & \text{anisotropic} \end{cases} \right) \right]$$

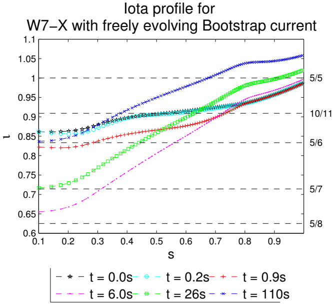
- Ampere's Law: $\mathbf{J} = \nabla \times \mathbf{B} + \nabla \times \delta \mathbf{B}$
- **Force**: $\mathbf{F} = \mathbf{J} \times \mathbf{B} - \nabla p = \mathbf{0}$
- **Energy minimization**: $W = \int_\Omega \left(\frac{B^2}{2\mu_0} + \frac{p}{\gamma - 1} \right) dV$

  via variational principle: $\delta W = -\int P^{ij} F_i F_j dV$

Table of Contents

Analysis domain is restricted to LCFS





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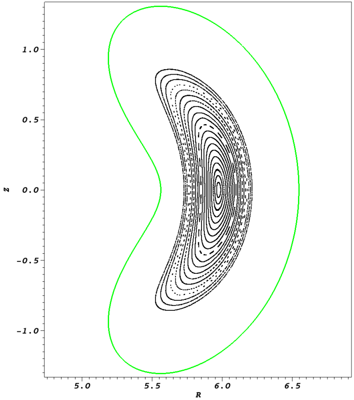
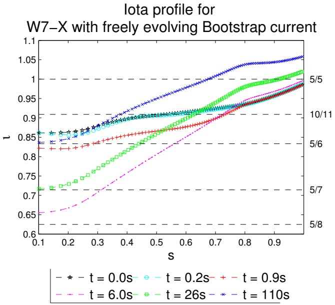
Limitation: Analysis domain

What happens outside the LCFS??





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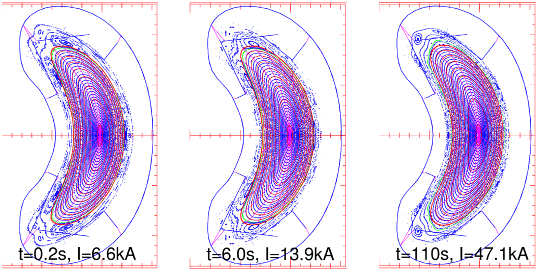

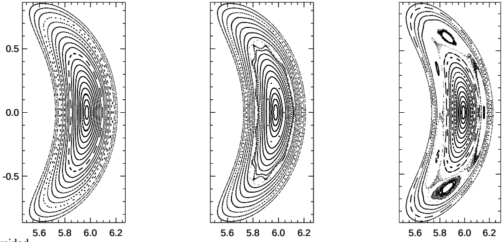

$t = 0.0s$ $t = 0.2s$ $t = 110s$

Comparison between results with VMEC+Extender (on top, Geiger et al., Contrib. Plasma Phis. 50 (8), 2010) and those of SIESTA for $\phi = 0$ (on the bottom).

What about self-consistency?

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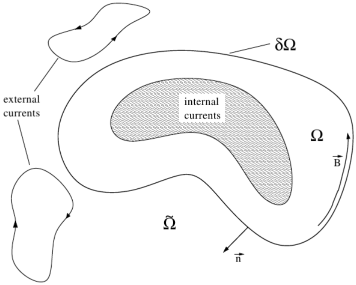
Table of Contents

Extension idea

- Increased domain $\rightsquigarrow$ Free boundary problem
- Virtual Casing: method applied already in the EXTENDER code.
- Would compete with PIES and HINT2 codes (they're slower, but already free boundary).

Extension idea: Virtual Casing Principle



Representation: Inside: Ω; Outside: $\tilde{\Omega}$; Surface: $\delta\Omega$.

From Drevlak *et al.*, Nuc. Fusion 45 (7), 2005.

Inside: $\mathbf{B}_{(\mathbf{r})} = \mathbf{B}^{\mathbf{J}_\Omega}_{(\mathbf{r})} + \mathbf{B}^{\mathbf{J}_{\tilde{\Omega}}}_{(\mathbf{r})}$

Def.: $\mathbf{B}^* = \begin{cases} \mathbf{B}_{(\mathbf{r})} & \mathbf{r} \in \Omega \\ 0 & \mathbf{r} \in \tilde{\Omega}, \end{cases}$

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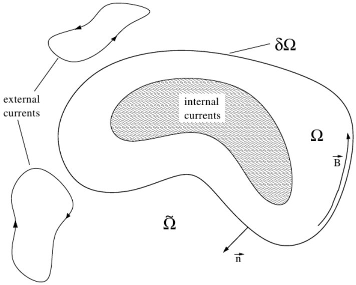

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$\mathbf{J}_{\delta\Omega(\mathbf{p})} = \frac{-1}{\mu_0} \mathbf{n} \times \mathbf{B}_{(\mathbf{p})}$

$\Rightarrow \mathbf{B}_{J_{\delta\Omega}} = -\mathbf{B}_{J_\Omega} \quad , \quad \mathbf{r} \in \tilde{\Omega}$

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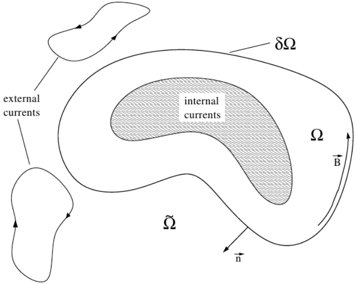

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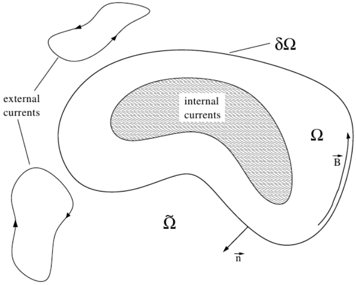
- Get current sheet $\mathbf{J}_{\delta\Omega}$
- Compute $\mathbf{B}_{\mathbf{J}_{\delta\Omega}}$ due to it
- Add it to the $\mathbf{B}_C$ due to the coils
- Solve the whole domain with SIESTA



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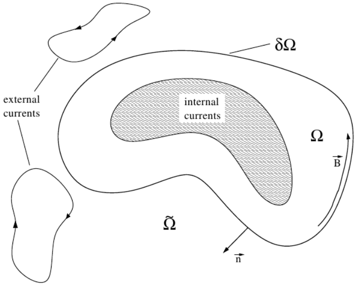
Extension idea: Virtual Casing Principle



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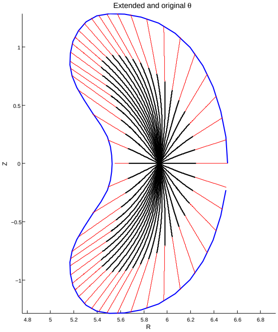
Where in $\tilde{\Omega}$ do we compute the field?

$\Longrightarrow$ Need to extend the numerical mesh

Requirements of the extension

- Continuity up to the first derivative (jacobian)
- No intersection of constant θ or ϕ lines



Done by extending Fourier coefficients up to the vessel

$$R_{(s)} = \sum_{m,n} R_{mn(s)} \cos(m\theta + n\phi)$$

$$Z_{(s)} = \sum_{m,n} Z_{mn(s)} \sin(m\theta + n\phi)$$

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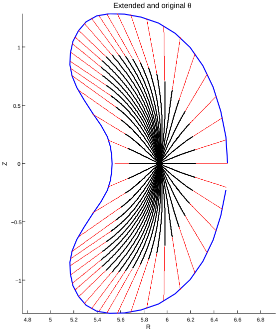

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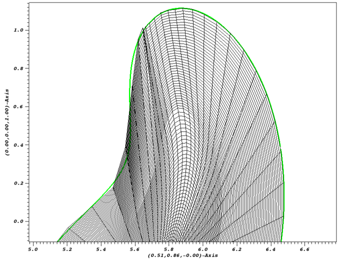

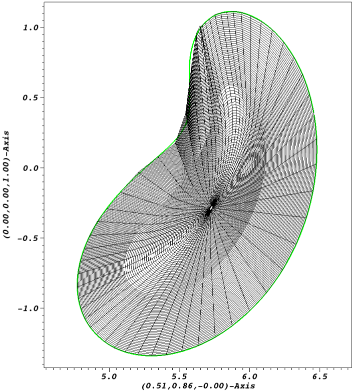

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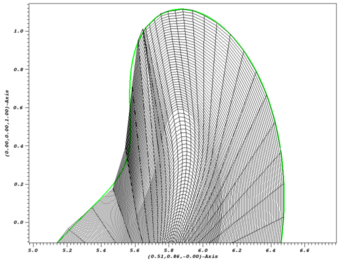

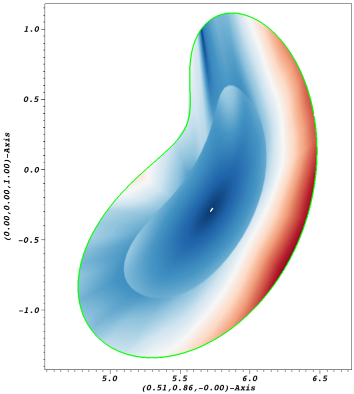

Table of Contents

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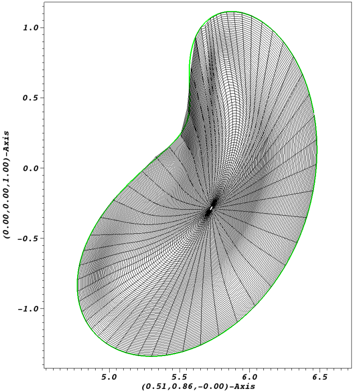
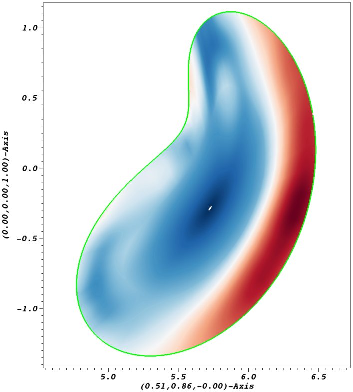
Present status

Numerical Mesh

Major problems in numerical mesh have been solved





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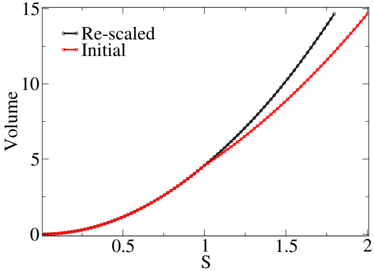
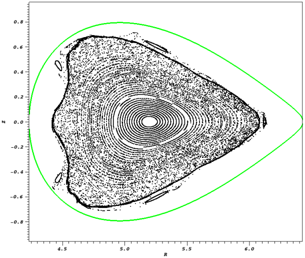
Definition of the radial [flux] label

$$\frac{ds'}{dV} = aV + b$$

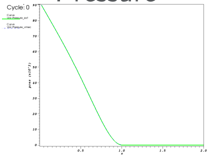


SIESTA needs a first approximation solution...

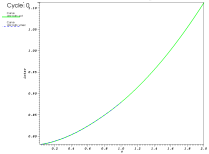
Field provided by the EXTENDER code



Pressure



ι-profile ¿?



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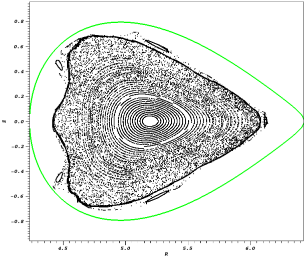

Converged ideally w/o pressure

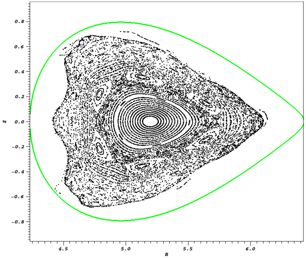

Table of Contents

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Pending issues / Future work

- Boundary condition needs to be redefined
- Revision of the resistive cases
- Include Virtual Casing tool within SIESTA.

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